RevMexAA (Serie de Conferencias), **35**, 76–77 (2009)

DETERMINING THE FRACTAL DIMENSION OF THE INTERSTELLAR MEDIUM

N. Sánchez,[1] E. J. Alfaro,[1] and E. Pérez[1]

RESUMEN

El medio interestelar parece tener una estructura fractal subyacente, la cual se puede caracterizar a través de su dimensión fractal (D_f). Sin embargo, diversos factores pueden afectar la determinación de D_f, tales como distorsiones causadas por la proyección, baja resolución de la imagen, la opacidad de la propia nube y una baja razón señal a ruido. En este trabajo utilizamos tanto nubes simuladas como mapas reales de nubes moleculares para estudiar estos efectos y poder estimar de una manera fiable la dimensión fractal. Los resultados indican de una forma autoconsistente que la dimensión fractal del medio interestelar está en el rango $2.6 \lesssim D_f \lesssim 2.8$, lo cual es significativamente más alto que el valor $D_f \simeq 2.3$ que usualmente se supone en la literatura.

ABSTRACT

The interstellar medium seems to have an underlying fractal structure, which can be characterized through its fractal dimension (D_f). However, several factors may affect the determination of D_f, such as distortions due to projection, low image resolution, opacity of the cloud, and low signal-to-noise ratio. Here we use both simulated clouds and real molecular cloud maps to study these effects in order to estimate D_f in a reliable way. Our results indicate in a self-consistent way that the fractal dimension of the interstellar medium is in the range $2.6 \lesssim D_f \lesssim 2.8$, which is significantly higher than the value $D_f \simeq 2.3$ usually assumed in the literature.

Key Words: ISM: clouds — ISM: individual (Ophiuchus, Orion, Perseus molecular cloud) — ISM: structure

1. INTRODUCTION

The fractal dimension (D_f) seems to be a simple but useful tool for characterizing the degree of complexity of the Interstellar Medium (ISM). Most of studies use the perimeter-area method to calculate the fractal dimension of the boundaries of projected molecular clouds ($D_{\rm per}$). The observational evidence can be summarized by saying that $D_{\rm per}$ has a more or less universal value around $\simeq 1.35$ (see Sánchez et al. 2005, and references therein). From this, it is usually assumed that the real 3-dimensional fractal dimension is $D_f = D_{\rm per} + 1 \simeq 2.35$ (e.g., Beech 1992), a value which moreover is consistent with turbulent diffusion in an incompressible fluid (Meneveau & Sreenivasan 1990). In this contribution we show that this simple relation between D_f and $D_{\rm per}$ is not valid. We study carefully the effects that different factors have on the determination of D_f in order to calculate in a highly reliable way its value for the ISM.

2. METHOD

Here we follow an empirical approach. We first simulate 3-dimensional fractal clouds with a given well-defined fractal dimension D_f (see details in Sánchez et al. 2005, 2006). Then we project the clouds to generate 2-dimensional maps. When doing this, we consider and analyze in detail the effects produced by the projection itself, image resolution, cloud opacity, and low signal-to-noise ratio (S/N). We take all these factors into consideration to develop algorithms to estimate fractal dimensions in a precise and accurate way. We use two different fractal estimators to corroborate the results and trends obtained: the perimeter-area-based dimension ($D_{\rm per}$) and the mass-size dimension (D_m).

3. FRACTAL DIMENSION ESTIMATION

Our main results can be summarized as follows.

• *Projection.* The projected perimeter dimension $D_{\rm per}$ decreases as the 3-dimensional fractal dimension D_f increases. This can be seen in Figure 1 for fractal clouds having dimensions in the range $2.0 \leq D_f \leq 2.9$. The interesting point here is that the observed average value $D_{\rm per} \simeq 1.35$ is more consistent with $D_f \simeq 2.6$ than with $D_f \simeq 2.3$.

• *Resolution.* We have decreased the resolution by hand and we have obtained that $D_{\rm per}$ decreases as resolution decreases. This is an expected result since as the pixel size is larger (worse resolution) the perimeter becomes smoother because the fractal features (the irregularities in the cloud contours) blend

[1]Instituto de Astrofísica de Andalucía, CSIC, Apdo. 3004, E-18080, Granada, Spain (nestor, emilio, eperez@iaa.es).

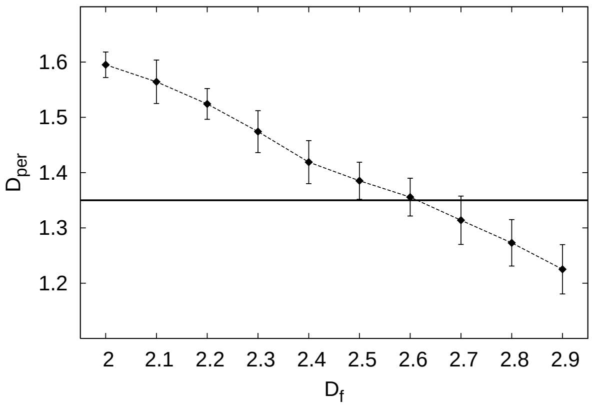

Fig. 1. Perimeter dimension $D_{\rm per}$ as a function of the fractal dimension D_f calculated for 50 random clouds. Each data point is the average value and the bars are the standard deviations. The horizontal solid line indicates the value $D_{\rm per} = 1.35$.

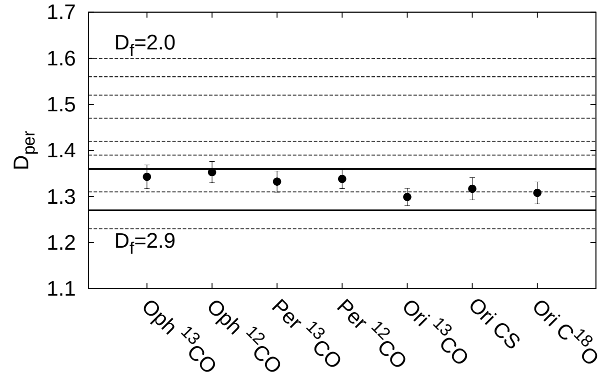

Fig. 2. Perimeter dimension $D_{\rm per}$ obtained for each molecular cloud map labeled in the horizontal axis. The horizontal lines indicate values calculated for simulated clouds with D_f values from 2.0 to 2.9 in increments of 0.1. The interval $2.6 \leq D_f \leq 2.8$ is emphasized with thick solid horizontal lines.

with each other. The results shown in Figure 1 correspond to "high resolution" images, i.e. cloud for which the size is $N_{\rm pix} \geq 400$ pixels. In this case we get $D_{\rm per} = 1.36 \pm 0.03$ for the case $D_f = 2.6$, but if the resolution is decreased to $N_{\rm pix} = 200$ then $D_{\rm per} = 1.28 \pm 0.02$.

• *Opacity.* We have generated cloud maps with different values for the total optical depth τ. We have obtained the important result that opacity does not affect the estimation of $D_{\rm per}$. This is because even for optically thick clouds, for which central and densest cores are difficult to detect, the fractal properties of the cloud are contained in the shape of the external contours. The situation is different for the mass dimension D_m because this estimator use information from all the cloud structure (mass vs. radius). This method (D_m) fails when $\tau \gtrsim 1$ but in any case it can be used without problems for optically thin regions.

• *Noise.* Very noisy maps can seriously affect the estimation of both $D_{\rm per}$ and D_m, artificially increasing the structure irregularities and therefore decreasing the final value of D_f. This effect can be minimized by smoothing the map before calculating D_f. We have shown that this previous step should be done in low-S/N maps only if the smoothing process maximizes S/N.

4. APPLICATION TO THE ISM

Taking all the above effects into account we have calculated the fractal dimension of Ophiuchus, Perseus, and Orion molecular clouds using emission maps in different lines. All the results are summarized in Figure 2, where we can see that the fractal dimension is always in the range $2.6 \lesssim D_f \lesssim 2.8$. The mass dimension method yielded very similar results (Sánchez et al. 2007).

These results are in perfect agreement with recent simulations of compressively driven turbulence in the ISM (Federrath et al. 2007), and this is an important point because, as pointed out by these authors, the ISM is far to be an incompressible fluid. Federrath et al. (2007) obtained $D_f \simeq 2.6 - 2.7$ in their standard simulations and they showed that D_f can be as small as ~ 2.4 only for the extreme case of high turbulent Mach numbers (of the order of 10). Another important point is that it has been argued that a fractal ISM with $D_f \simeq 2.3$ could account for some observed cloud properties (e.g., Elmegreen & Falgarone 1996), but we have also shown that $D_f \simeq 2.6$ is roughly consistent with the average properties of the ISM, in particular the mass and size distributions (Sánchez et al. 2006). In summary, it seems that $D_f \simeq 2.7 \pm 0.1$ for the ISM. The relevance of this relatively high fractal dimension value has to be analyzed, mainly concerning the physical processes involved in the structure of the ISM.

REFERENCES

Beech, M. 1992, Ap&SS, 192, 103
Elmegreen, B. G., & Falgarone, E. 1996, ApJ, 471, 816
Federrath, C., Klessen, R. S., & Schmidt, W. 2007, ApJ, submitted (arXiv:0710.1359)
Meneveau, C., & Sreenivasan, K. R. 1990, Phys. Rev. A, 41, 2246
Sánchez, N., Alfaro, E., & Pérez, E. 2005, ApJ, 625, 849
__________. 2006, ApJ, 641, 347
__________. 2007, ApJ, 656, 222