RevMexAA (Serie de Conferencias), **35**, 19–20 (2009)

ANALYTICAL MODEL FOR MARS CRATER-SIZE FREQUENCY DISTRIBUTION

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RESUMEN

Presentamos un modelo matemático que reproduce las características esenciales de la distribución de número de cráteres, N *vs.* diámetro, D, de los aproximadamente 42,000 cráteres contenidos en el Catálogo de Barlow. El modelo se deriva a partir de premisas sencillas que permiten relacionar la población de cráteres del presente con la producción de cráteres a través de diferentes épocas. En el análisis se toma en cuenta la reducción en el número de cráteres debido a procesos de obliteración y erosión, y esto sirve para explicar de manera simple el porqué de la existencia de diferentes pendientes en la curva empírica log(N) *vs.* log(D). La vida media para cráteres en Marte se deduce, y se demuestra que este resultado es consistente con la correspondiente determinación de la vida media de cráteres de la Tierra. Se presentan argumentos que explican que esta consistencia es consecuencia de que la vida media de un cráter es proporcional a su volumen. Otra conclusión importante del modelo es que el flujo de impactos, ϕ, está dado por $\phi \propto 1/D^{4.3}$. Esto llama la atención, ya que el exponente 4.3 es mayor que lo previamente reconocido, e implica un exponente similar para la distribución de número *vs.* diámetro de impactores.

ABSTRACT

We present a theoretical and analytical curve that reproduces essential features of the frequency distributions *vs.* diameter of the 42,000 impact craters contained in Barlow's Mars Catalog. The model is derived using reasonable simple assumptions that allow us to relate the present craters population with the craters population at each particular epoch. The model takes into consideration the reduction of the number of craters as a function of time caused by their erosion and obliteration, and this provides a simple and natural explanation for the presence of different slopes in the empirical log-log plot of number of craters (N) *vs.* diameter (D). A mean life for martians craters as a function of diameter is deduced, and it is shown that this result is consistent with the corresponding determination of craters mean life based on Earth data. Arguments are given to suggest that this consistency follows from the fact that a crater mean life is proportional to its volumen. It also follows that in the absence of erosions and obliterations, when craters are preserved, we would have $N \propto 1/D^{4.3}$, which is a striking conclusion, since the exponent 4.3 is larger than previously thought. Such an exponent implies a similar slope in the extrapolated impactors size-frequency distribution.

Key Words: planets and satellites: Mars

1. INTRODUCTION

The present impact crater size frequency distribution, N, is the result, on one hand, of a rate of crater formation, ϕ, and, on the other hand, the elimination of craters, as time goes by, due to effects like erosion and obliteration. The above relation is described by the following differential equation:

$$dN = \phi\, dt - NC\, dt\,, \tag{1}$$

where NC is the rate of crateres elimination. The simplest model, which follows from the integration of equation (1) that fit the Barlow's Mars impact craters catalog data (Barlow 1988) is (see Figure 1)

$$N(D) = \frac{\phi(D)}{C(D)}\left[1 - \exp\{-C\tau_f\}\right]\,, \tag{2}$$

$$\phi(D) = \frac{3.55\times 10^{9}}{D^{4.3}\,\tau_f}\,, \tag{3}$$

$$C(D) = \frac{2.48\times 10^{4}}{D^{2.5}\,\tau_f}\,, \tag{4}$$

where τ_f is the total time of integration. Note that, since a given initial population of craters N_0 decreases according to the formula:

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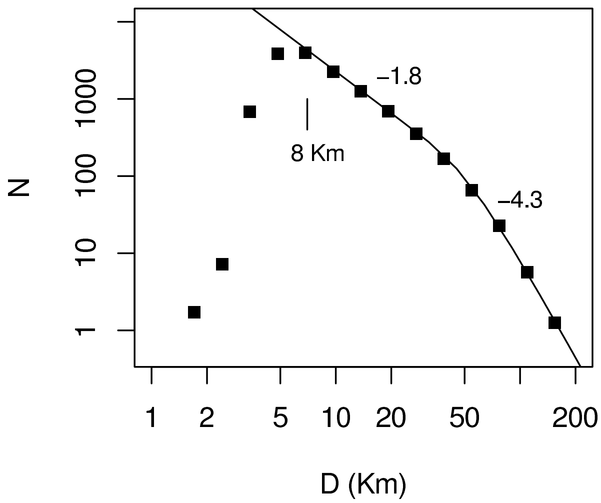

Fig. 1. Comparison of theoretical model with the empirical log-log plot of number of craters (N) *vs.* diameter (D) in Mars. Craters with $D < 8$ km are undercounted.

$$N = N_0 \, e^{-Ct} , \tag{5}$$

then the mean life (T) of crateres of diameter D is $T = 1/C$. Studies by (Garvin 2002) show that Mars craters volumen, V, behaves statistically as $V \propto D^{2.5}$, and thus we have that the mean life $T \propto V$, which is a result that is intuitively appealing.

We see from Figure 1 and equations (2–4) that, for $D > 8$km, the Martian Crater-Size Frequency Distribution is essentially described by two straigh lines of slopes −1.8 and −4.3. The −1.8 slope corresponds to a steady state, where the rate of production of craters, ϕ, is equal to the rate of elimination NC. On the other hand, the −4.3 slope is pristine, since for this larger crateres $N = \phi \, \tau_f$, and elimination of crateres plays no role. The slope −1.8 is also observed in the crater-size frequency distribution of Earth and the Moon, which seems to indicate that on Mars, Earth and the Moon we have $T \propto V \propto D^{2.5}$.

We can test in our planet the validity of the relation $T \propto D^{2.5}$, since we know the crater age, t, and then, using equation (5), we are able to show that

$$\log(\bar{D}) = \frac{1}{p} \log \; t + \text{const} , \tag{6}$$

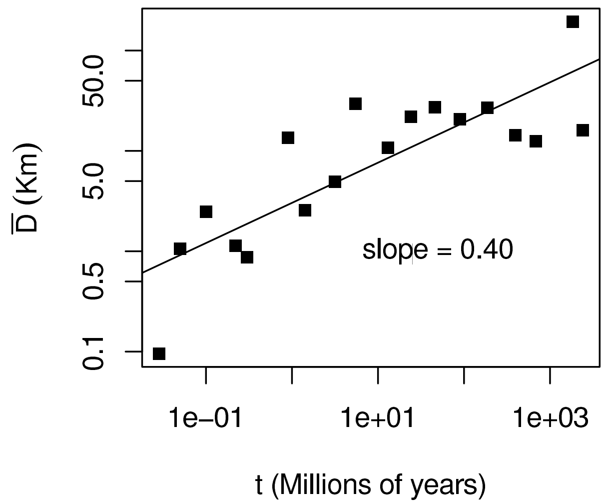

Fig. 2. Average diameter ($\bar{D}$) *vs.* age (t) for terrestrial craters. The bin size increases as $2^{n/2}$. The slope of the straight line best fit (0.40) correspond to $p = 2.5$.

where $\bar{D}$ is the average diameter of craters of age t and we are assuming that $T = 1/C \propto D^p$. In Figure 2 Figure 2 we plot $\log(\bar{D})$ *vs.* $\log(t)$ and find that the best straight line fitting slope, $1/p$, gives indeed $p = 2.5$.

The large exponent 4.3 in the impact flux ϕ have interesting implications for the corresponding impactors size *vs.* number population. For instance, the reasonable assumption that the diameter of an impactor, D_i, is nearly proportional to D imply that $N \propto 1/D_i^{4s}$, $s \approx 1$. Observations of diameter-frequency distribution for asteroids (see for example Poveda 1999; Ivezić 2002) show that a large exponent describe the asteroid population, and our results are in agreement with these observations.

We are grateful to Dr. Nadine Barlow for helpful suggestions.

REFERENCES

Barlow, N. G. 1988, Icarus, 75, 285

Garvin, J. B. 2002, Lunar Planet. Sci., XXXIII, 1255

Poveda, A., et al. 1999, Planet. Space Sci., 47, 679

Ivezić, Z., et al. 2002, Proc. SPIE, 4836, 98