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THE EULER CHARACTERISTIC AS A MEASURE OF THE TOPOLOGY OF COSMIC REIONIZATION

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RESUMEN

Presentamos una introducción al uso de la Característica de Euler en simulaciones a gran escala de la época de la reionización cósmica. Para caracterizar la topología del campo de la fracción de ionización, hemos calculado la evolución de su Característica de Euler. Nuestros resultados indican que la evolución de la topología durante la primera mitad de la reionización cósmica, es consistente con la reionización de dentro hacia afuera de un campo de densidad (log) Gaussiano.

ABSTRACT

We present here an introduction to the use of the Euler Characteristic in large-scale simulations of cosmic reionization. To characterize the topology of the ionization fraction field, we calculate the evolution of its Euler Characteristic. We find that the evolution of the topology during the first half of reionization is consistent with inside-out reionization of a (log) Gaussian density field.

Key Words: cosmology: theory — H II regions — methods: numerical

1. INTRODUCTION TO COSMIC REIONIZATION

The epoch of cosmic reionization (EoR) begins with the formation of the first sources of light and ends at the time where most of the intergalactic medium (IGM) is ionized. There are mainly two observations constraining the EoR: The enhanced polarization in the cosmic microwave background (CMB) radiation at large angular scales (resulting from Thompson scattering of the photons off free electrons) and the absence of the Gun-Peterson through in high-redshift quasar spectra (no complete absorption of quasar light bluewards of the Lyα(H) emission line).

If cosmic reionization would be a step function in time, the redshift z of transition can be calculated from the polarization power spectrum of the CMB. The value according to the 5 year WMAP observations is $z \approx 10.9 \pm 1.4$ (Komatsu etal. 2009). The quasar spectra obtained within the Sloan Digital Sky Survey (SDSS) indicate a low, but rapidly rising neutral fraction around $z \sim 6$ (e.g. Willott et al. 2007). The combination of these measurements suggests an extended EoR.

Upcoming 21cm experiments (GMRT[5], LOFAR[6], MWA[7], to name a few), will be able to observe the EoR directly.

1.1. *Simulating Cosmic Reionization*

In a typical EoR simulation, there are five steps:
(1) N-body dark matter (DM) only simulation to get the density field as a function of redshift,
(2) assume gas follows DM → gas-density field,
(3) identify halos,
(4) assign luminosity to halos,
(5) trace ionizing radiation through IGM & ionize.

For step (1), we use the CubeP^{3}M code which was developed from the PMFAST code (Merz, Pen, & Trac 2005), see Iliev et al. (2008). The simulation uses 1024^3 particles and 2048^3 cells, which imply a particle mass of $5.1 \times 10^6\ M_\odot$. The minimum resolved halo mass is $10^8\ M_\odot$ which is approximately the minimum mass of halos able to cool by atomic hydrogen cooling. The cosmological parameters used were for a flat ΛCDM universe with $(\Omega_m, \Omega_b, h, n, \sigma_8) = (0.27, 0.044, 0.7, 0.96, 0.8)$.

Step (5) is performed using the C^2-Ray method (Mellema et al. 2006) on a uniform rectilinear grid

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[5]Giant Metrewave Telescope, `http://gmrt.ncra.tifr.res.in`.

[6]Low Frequency Array, `www.lofar.org`.

[7]Murchison Widefield Array, `www.mwatelescope.org`.

containing 256^3 grid cells. We assign ionizing luminosities via a constant light-to-(baryonic)mass ratio. This ratio is larger for low mass halos: $\sim 1.5 \times 10^{58}$ ionizing photons per $M_\odot$ for halo masses $< 10^9\ M_\odot$ and $\sim 1 \times 10^{57}$ for those above $10^9\ M_\odot$. This is motivated for example by the different composition in terms of stellar populations and by different escape fractions due to more clumping and more dust in in high mass halos. Additionally, we suppress sources in low mass halos if they are located in regions with an ionization fraction larger 10%, for details see Iliev et al. (2007). The simulation analyzed here is 53Mpc_g8.7_130S in Friedrich et al. (2011).

2. INTRODUCTION TO THE EULER CHARACTERISTIC

The Euler Characteristic (henceforth V_3) is one of four topological invariant Minkowski functionals in three dimensions. For bodies defined by closed surfaces, V_3 can be defined as V_3 = Number of parts- number of tunnels + number of cavities. With this definition, we follow Schmalzing & Buchert (1997) whose algorithm we are using. With this definition, V_3 of a football is 1–0+1 = 2; V_3 of a solid torus is 1–1+0=0 and V_3 of a solid sphere is 1–0+0=1. The Euler Characteristic has been used in many fields of physics, also in cosmology to characterize the topology of large scale structure (e.g. Mecke, Buchert, & Wagner 1994).

2.1. V_3 *of a Gaussian random field (GRF)*

To calculate V_3 of a GRF, we need to divide the continuous field into background (BC) and foreground cells (FC). One can use a threshold value $x_{\rm th}$ to do so: all points with values below $x_{\rm th}$ are BC, the remaining are FC; the FC define the bodies. Choosing a low $x_{\rm th}$ results in many disconnected background objects (holes), choosing a very high threshold value, results in many disconnected bodies. In Figure 1 (left panel) we plot $V_3(x_{\rm th})$ of a GRF. The topology obtained at low $x_{\rm th}$ is called swiss-cheese-topology, the complex topology obtained at intermediate $x_{\rm th}$ is called sponge-topology and the topology at high $x_{\rm th}$ is called meatball-topology.

3. V_3 OF THE SIMULATED IONIZATION FRACTION FIELD DURING EOR

Our goal is to investigate the evolution of the topology of the ionization fraction field in our EoR simulations as a function of redshift, or equivalently, average ionization fraction $\langle x \rangle$. Therefore, we need to choose a $x_{\rm th}$ below which cells count as neutral (BC) and above which cells count as ionized (FC).

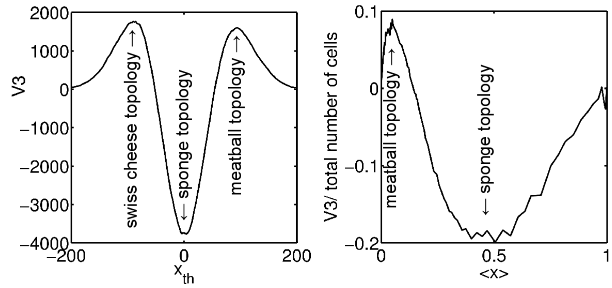

Fig. 1. Left: V_3 of a GRF as a function of $x_{\rm th}$; right: $V_3(\langle x \rangle)$ of the ionization fraction field at $x_{\rm th} = 0.5$. The small fluctuations at high $\langle x \rangle$ are due to our implementation of photons traveling longer distances than a simulation box-length, they are not important here.

In the case of sharp ionization fronts, which are to be expected in this simulation since only black body sources are included, V_3 should be independent of the chosen $x_{\rm th}$. Due to finite resolution, and problems related to edge- and corner- connections of BCs and FCs, there is actually some dependence on $x_{\rm th}$. Between $x_{\rm th} = 0.2$ and 0.6, the evolution of V_3 is largely independent on the actual choice of $x_{\rm th}$; in the right panel of Figure 1 we show $V_3(\langle x \rangle)$ at $x_{\rm th} = 0.5$.

V_3 of the ionization field evolves in the first half as V_3 of a GRF but at high $\langle x \rangle$ there is no rise to positive values followed by a decrease. This shows that if there exists a monotonic steady functional relation between density and time of ionization it does so only until a certain density: low-density areas (the voids in the density field) are ionized "before their time" and do not serve as positive contributions to V_3 in the ionization field. This is a feature of what is called inside-out reionization: the H II regions start in high density regions but do eventually break out in the voids.

The Euler Characteristic can be used to test the effect of different source properties on the topology of the ionization fraction field, see Friedrich et al. (2011).

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