

## THE BREAKDOWN OF CLASSICAL GRAVITY?

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### RESUMEN

Bajo el supuesto de que la ley de la gravedad de Newton y la Relatividad General (RG) son válidas a todas las escalas, surge la hipótesis de la existencia de materia oscura para poder explicar la dinámica de los sistemas astronómicos a escalas galácticas y extragalácticas. Otra posibilidad, que no involucra la existencia de ningún componente exótico, es modificar la ley de la gravedad a escalas donde la aceleración es menor que  $a_0 = 1.2 \times 10^{-8} \text{ cm s}^{-2}$ . Las órbitas de estrellas binarias abiertas nos ofrecen un experimento decisivo en este debate. Para sistemas binarios la aceleración cae por debajo de  $a_0$  a separaciones mayores que 7000 au, en este punto las teorías de gravedad modificada predicen velocidades constantes que no dependen de la separación, en contraste con la tercera ley de Kepler que predice velocidades que decrecen con la raíz de la separación. Realizamos este experimento con una muestra de sistemas binarios con velocidades relativas y separaciones mayores que  $10^4$  au. Obtenemos como resultado un límite superior constante para la velocidades relativas de las estrellas binarias abiertas independiente de la separación, en contradicción con la tercera ley de Kepler y en acuerdo con lo predicho por las teorías de gravedad modificada.

### ABSTRACT

Assuming Newton's gravity and General Relativity (GR) to be valid at all scales leads to the dark matter hypothesis as a requirement demanded by the observed dynamics and measured baryonic content at galactic and extragalactic scales. Alternatively, modified gravity scenarios where a change of regime appears at acceleration scales  $a < a_0$  have been proposed. This modified regime at  $a < a_0$  will generically be characterised by equilibrium velocities which become independent of distance. Here we identify a critical test in this debate and we propose its application to samples of wide binary stars. Since for  $1 M_\odot$  systems the acceleration drops below  $a_0$  at scales of around 7000 au, a statistical survey of wide binaries with relative velocities and separations reaching  $10^4$  au and beyond should prove useful to the above debate. We apply the proposed test to the best currently available data. Results show a constant upper limit to the relative velocities in wide binaries which is independent of separation for over three orders of magnitude, in analogy with galactic flat rotation curves in the same  $a < a_0$  acceleration regime. Our results are suggestive of a breakdown of classical gravity beyond  $a \approx a_0$  scales, in accordance with generic predictions of modified gravity theories designed not to require any dark matter at galactic scales and beyond.

*Key Words:* binaries: general — cosmology: theory — dark matter — gravitation

### 1. INTRODUCTION

Over the past years the dominant explanation for the large mass to light ratios inferred for galactic and meta-galactic systems, that these are embedded within massive dark matter halos, has begun to be challenged. Direct detection of the dark matter particles, in spite of decades of extensive and dedicated searches, remains lacking. Numerous alternative theories of gravity have appeared (e.g. TeVeS of Bekenstein 2004; F(R) theories, e.g. Sobouti 2007; Capozziello et al. 2007) mostly grounded on geometrical extensions to GR, and leading to laws of gravity which in the low acceleration regime, mimic

the MODified Newtonian Dynamics (MOND) fitting formulas, e.g. Milgrom (2010). Similarly, Mendoza et al. (2011), have explored MOND not as a modification to Newton's second law, but as a modified gravitational force law in the Newtonian regime, finding a good agreement with observed dynamics across galactic scales without requiring dark matter. Recently, Bernal et al. (2011) have constructed an  $f(R)$  extension to general relativity which in the low velocity limit converges to the above approach.

A generic feature of all such modified gravity schemes is the appearance of an acceleration scale,  $a_0$ , above which classical gravity is recovered, and below which the dark matter mimicking regime appears. This results in a general prediction; all systems where  $a \gg a_0$  should appear as devoid of dark

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matter, and all systems where  $a \ll a_0$  should appear as dark matter dominated. It is interesting that no  $a \gg a_0$  system has ever been detected where dark matter needs to be invoked. On the other hand, the latter condition furnishes a testable prediction in relation to the orbits of wide binaries. For test particles in orbit around a  $1 M_\odot$  star, in circular orbits of radius  $s$ , the acceleration is expected to drop below  $a_0 \approx 1.2 \times 10^{-10} \text{ m s}^{-2}$  for  $s > 7000 \text{ au} = 3.4 \times 10^{-2} \text{ pc}$ . This provides a test for the dark matter/modified theories of gravity debate; the relative velocities of components of binary stars with large physical separations should deviate from Kepler's third law. Seen as an equivalent Newtonian force law, beyond  $s \approx 7000 \text{ au}$  the gravitational force should gradually switch from the classical form of  $F_N = GM/s^2$  to  $F_{MG} = (GMa_0)^{1/2}/s$ , and the orbital velocity,  $V^2/s = F$ , should no longer decrease with separation, but settle at a constant value, dependent only on the total mass of the system through  $V = (GMa_0)^{1/4}$ . In a recent study giving results in agreement with this scenario, Hernández & Jiménez (2012), have constructed gravitational equilibrium dynamical models for a sample of eight Galactic globular clusters for which radial profiles of the projected velocity dispersion have been observed to stop decreasing along Keplerian expectations, and to level off at constant values beyond the radii where  $a < a_0$ . Observationally, they find that the asymptotic values of the velocity dispersion profiles are consistent with scaling with the fourth root of the total masses, a Tully-Fisher relation for globular clusters. Under modified gravity theories, binary stars with separations beyond  $7000 \text{ au}$  should exhibit "flat rotation curves" and a "Tully-Fisher relation", as galactic systems in the same acceleration regime do. We apply this test to the binaries of two recent catalogues containing relative velocities and separations. The two catalogues are entirely independent in their approaches. The first one, Shaya & Olling (2011), comprises 280 systems from the *Hipparcos* satellite, while the second, Dhital et al. (2010), identifies 1,250 ones from the Sloan Digital Sky Survey (SDSS). Details can be found in Hernández et al. (2012).

## 2. RELATIVE VELOCITY DISTRIBUTIONS FOR WIDE BINARIES UNDER NEWTONIAN EXPECTATIONS

The Newtonian prediction for the relative velocities of the two components of binaries having circular orbits, when plotted against the binary physical separation,  $s$ , is for a scaling of  $\Delta V \propto s^{-1/2}$ , essentially following Kepler's third law, provided the range of

masses involved were narrow. In a relative proper motion sample however, only two components of the relative velocity appear, as velocity along the line of sight to the binary leads to no proper motion. Thus, orbital projection plays a part, with systems having orbital planes along the line of sight sometimes appearing as having no relative proper motions. A further effect comes from any degree of orbital ellipticity present; it is hence clear that the trend for  $\Delta V \propto s^{-1/2}$  described above, will only provide an upper limit to the distribution of projected  $\Delta V$  vs.  $s$  expected in any real observed sample, even if only a narrow range of masses is included. One should expect a range of measured values of projected  $\Delta V$  at a fixed observed projected  $s$ , all extending below the Newtonian limit, which for equal mass binaries in circular orbits gives  $\Delta V_N = 2 \left( \frac{GM}{s} \right)^{1/2}$ .

Further, over time, the orbital parameters of binaries will evolve due to the effects of Galactic tidal forces and dynamical encounters with other stars in the field, specially in the case of wide binaries. To first order, one would expect little evolution for binaries tighter than the tidal limit of  $1.7 \text{ pc}$ , and the eventual dissolution of wider systems. A very detailed study of all these points has recently appeared, Jiang & Tremaine (2010). These authors numerically follow populations of 50,000  $1 M_\odot$  binaries in the Galactic environment, accounting for the evolution of the orbital parameters of each due to the cumulative effects of the Galactic tidal field at the Solar radius. Also, the effects of close and long range encounters with other stars in the field are carefully included, to yield a present day distribution of separations and relative velocities for an extensive population of wide binaries, under Newtonian Gravity.

It is found that when many wide binaries cross their Jacobi radius, the two components remain fairly close by in both coordinate and velocity space. Thus, in any real wide binary search a number of wide pairs with separations larger than their Jacobi radii will appear. Finally, Jiang & Tremaine (2010), obtain the RMS one-dimensional relative velocity difference,  $\Delta V_{1D}$ , projected along a line of sight, for the entire populations of binaries dynamically evolved over  $10 \text{ Gyr}$  to today, as plotted against the projected separation on the sky for each pair. The expected Keplerian fall of  $\Delta V_{1D} \propto s^{-1/2}$  for separations below  $1.7 \text{ pc}$  is obtained, followed by a slight rise in  $\Delta V_{1D}$  as wide systems cross the Jacobi radius threshold.  $\Delta V_{1D}$  then settles at RMS values of  $\approx 0.1 \text{ km s}^{-1}$ . This represents the best currently available estimate of how relative velocities should scale with projected separations for binary

stars (both bound and in the process of dissolving in the Galactic tides) under Newtonian gravity. We see that all we need is a large sample of relative proper motion and binary separation measurements to test the Newtonian prediction for the RMS values of the one-dimensional relative velocities of Jiang & Tremaine (2010), and to contrast the  $\Delta V_N \propto s^{-1/2}$  and the  $\Delta V_{MG} = \text{constant}$  predictions for the upper envelope of the  $\Delta V$  vs.  $s$  distributions.

### 3. OBSERVED WIDE BINARY SAMPLES

In the Shaya & Olling (2011) catalogue wide binaries are identified by assigning a probability above chance alignment to the systems by carefully comparing to the underlying background (and its variations) in a 5 dimensional parameter space of proper motions and spatial positions. We keep only binaries with a probability of non-chance alignment greater than 0.9. The binary search criteria used by the authors requires that the proposed binary should have no near neighbours; the projected separation between the two components is thus always many times smaller than the typical interstellar separation. We use the reported distances to the primaries, where errors are smallest, to calculate projected  $\Delta V$  and projected  $s$  from the measured  $\Delta\mu$  and  $\Delta\theta$  values reported. Although the use of *Hipparcos* measurements guarantees the best available quality in the data, we have also removed all binaries for which the final signal to noise ratio in the relative velocities was lower than 0.3.

We are left with a sample of 280 binaries, having distances to the Sun within  $6 < d < 100$  in pc. The data show a perfectly flat upper envelope in a  $\Delta V$  vs. projected  $s$  (Hernández et al. 2012). The average signal to noise ratio for the data is 1.7, with an average error on  $\Delta V$  of  $0.83 \text{ km s}^{-1}$ , which considering a  $2\sigma$  factor from the top of the distribution to the real underlying upper limit for the sample, results in  $3 \text{ km s}^{-1}$  as our estimate of the actual physical upper limit in  $\Delta V$ , Figure 1.

The Sloan low mass wide pairs catalogue (SLoW-PoKES) of Dhital et al. (2010) contains a little over 1,200 wide binaries with relative proper motions for each pair, distances and angular separations. Also, extreme care was taken to include only physical binaries, with a full galactic population model used to exclude chance alignment stars using galactic coordinates and galactic velocities, resulting in an estimate of fewer than 2% of false positives. This yields only isolated binaries with no neighbours within many times the internal binary separation. Again, we use the reported distances to the primaries to cal-

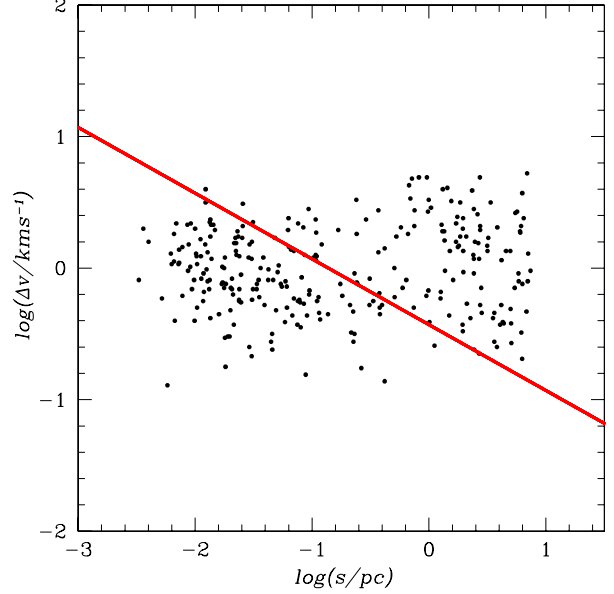


Fig. 1. The figure shows projected relative velocities for each binary pair and projected separations, for wide binaries from the Shaya & Olling (2011), *Hipparcos* catalogue having a probability of being the result of chance alignment  $< 0.1$ . The average value for the signal to noise ratio for the sample shown is 1.7. The upper limit shows the flat trend expected from modified gravity theories, at odds with Kepler's third law, shown by the  $s^{1/2}$  solid line.

culate projected  $\Delta V$  and projected  $s$  from the measured  $\Delta\mu$ ,  $\Delta\theta$  and  $d$  values reported by Dhital et al. (2010), to obtain a sample of 417 binaries.

The upper envelope of the distribution of  $\Delta V$  from this catalogue does not comply with Kepler's third law. As was the case with the *Hipparcos* sample, the upper envelope describes a flat line, as expected under modified gravity schemes. The average signal to noise in  $\Delta V$  for the Dhital et al. (2010) catalogue is 0.48, with an average error on  $\Delta V$  of  $12 \text{ km s}^{-1}$ , which considering a  $2\sigma$  factor from the top of the complete distribution to the real underlying upper limit gives the same  $3 \text{ km s}^{-1}$  as obtained for the Shaya & Olling (2011) *Hipparcos* catalogue. Figure 2 shows the RMS value of the one-dimensional relative velocity difference for both of the samples discussed. The error bars give the error propagation on  $\Delta\mu$  and  $d$ . We construct  $\Delta V_{1D}$  by considering only one coordinate of the two available from the relative motion on the plane of the sky. Thus, each binary can furnish two  $\Delta V_{1D}$  measurements, which statistically should not introduce any bias. Indeed, using only  $\Delta\mu_l$  or only  $\Delta\mu_b$  or both for each binary,

yields the same mean values for the points shown. The small solid error bars result from considering an enlarged sample where each binary contributes two  $\Delta V_{1D}$  measurements, while the larger dotted ones come from considering each binary only once, and do not change if we consider only  $\Delta\mu_l$  or only  $\Delta\mu_b$ . The series of small  $\log(s)$  interval data are for the *Hipparcos* catalogue of Shaya & Olling (2011), while the two broader crosses show results for the Dhital et al. (2010) SDSS sample.

The solid curve is the Newtonian prediction of the full Galactic evolutionary model of Jiang & Tremaine (2010), for binaries, both bound and in the process of dissolving. Note that the results of this simulation deviate from Kepler's law both for  $s$  larger than the Newtonian Jacobi radius of 1.7 pc, and also for separations  $s < 1.7$  pc. Even considering the large error bars, where each binary contributes only one  $\Delta V_{1D}$  value, we see eight points lying beyond  $1\sigma$ , making the probability of consistency between this prediction and the observations of less than  $(0.272)^8 = 3 \times 10^{-5}$ .

We obtain a constant RMS value for  $\Delta V_{1D}$  of  $1 \text{ km s}^{-1}$ , in qualitative agreement with expectations from modified gravity schemes. The vertical line marks  $a = a_0$ ; we see the data departing from the Newtonian prediction outwards of this line, and not before. It is crossing the  $a_0$  threshold, not the tidal radius, what determines the onset of the discrepancy. The two independent catalogues, each using different sets of selection criteria, each perhaps subject to its own independent systematics, are consistent with the same result, a constant horizontal upper envelope for the distribution of relative velocities on the plane of the sky at an intrinsic value of  $3 \text{ km s}^{-1} \pm 1 \text{ km s}^{-1}$ , extending over 3 orders of magnitude in  $s$ , with a constant RMS  $\Delta V_{1D}$  value consistent with  $1 \text{ km s}^{-1} \pm 0.5 \text{ km s}^{-1}$ . This supports the interpretation of the effect detected as the generic prediction of modified gravity theories.

#### 4. CONCLUSIONS

We identify a critical test in the classical gravity/modified gravity debate, using the relative velocities of wide binaries with separations in excess of 7000 au, as these occupy the  $a < a_0$  regime. We present a first application of this test using the best currently available data. Results show constant relative RMS velocities for the binary stars in question. This is quantitatively inconsistent with detailed predictions of Newtonian dynamical models for binaries evolving in the local galactic environment. Our re-

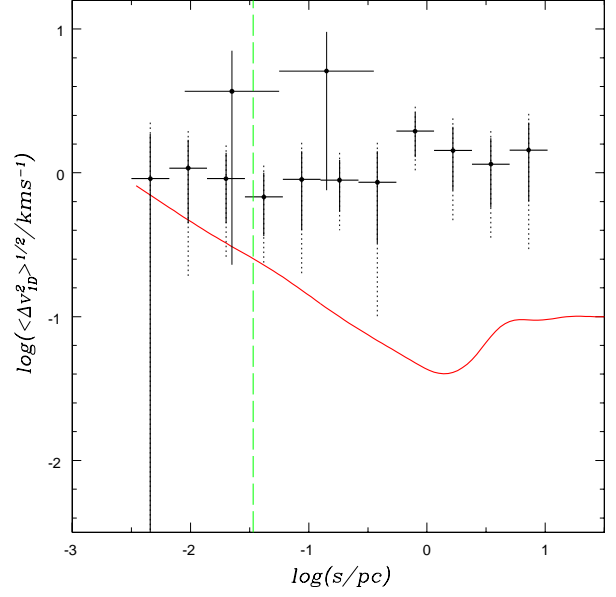


Fig. 2. The solid curve gives the RMS values for one dimensional projected relative velocities as a function of projected separations, for the dynamical modelling of large populations of wide binaries evolving in the Galactic environment, taken from Jiang & Tremaine (2010). The same quantity for the data from the catalogues analysed is given by the points with error bars; those with narrow  $\log(s)$  intervals being from the *Hipparcos* sample of Shaya & Olling (2011), and those two with wide  $\log(s)$  intervals coming from the SDSS sample of Dhital et al. (2010). The vertical dashed line marks the  $a = a_0$  threshold.

sults are qualitatively in accordance with generic modified gravity models which explain galactic dynamics in the absence of dark matter.

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