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INFORMATION AND DECISION THEORY AS APPLIED TO ASTRONOMY: THE CASE OF ASTROMETRY AND PHOTOMETRY

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RESUMEN

El límite de mínima varianza de Cramér-Rao se utiliza para establecer una cota para la máxima precisión astrométrica que se puede obtener con un detector CCD dadas las propiedades de la fuente, las características del detector, y las condiciones de observación.

ABSTRACT

Applying results from information and decision theory, in particular the Cramér-Rao lower variance bound, we place limits on the maximum astrometric precision attainable by a CCD detector given the properties of the source, the characteristics of the detector, and the observing conditions.

Key Words: astrometry — methods: analytical — methods: data analysis — methods: statistical

1. INFORMATION AND DECISION THEORY

Given a set of independent measurements for which we have good reasons to believe that they follow an underlying physical process determined by one (or more) parameters, a basic methodological question is: What is the minimum variance (maximum precision) attainable in the determination of such a parameter(s) given our data? This question, which is of paramount importance when we perform (or design) any experiment and, in particular, of outmost relevance in observational astronomy, is at the realm of a whole branch of multidisciplinary research that encompasses computer science, mathematics and statistics, and electrical engineering known as "Information Theory"[3].

The subject of information and decision theory is believed to have been founded by Claude E. Shannon, through the publication in 1948 of his seminal paper entitled "A Mathematical Theory of Communication" (Shannon 1948), whose main focus is the engineering problem of the transmission of information over a noisy channel. The results of Shannon and all the subsequent and substantial research in the field, fueled by the interest of the telecommunications community, has led to many sophisticated estimation techniques (see, e.g., Cover and Thomas (2006)), only a few of which have been applied to astronomy, in particular because most of the astronomy degree-granting institutions do not incorporate advanced modern statistics in their curricula, nor are these topics regularly covered in courses of observational astronomy at the under-graduate or graduate level.

Since 2013 we have begun a collaboration between the Astronomy and Electrical Engineering departments of our university, through the Information and Decision Systems Group (see `http://www.ids.uchile.cl/`), applying some of the results of this field in astronomy, in particular in astrometry and photometry using CCD detectors. In this poster we give a summary of our research so far, which is mainly based on the results presented by Méndez et al. (2013, 2014), where full details and discussions are presented.

2. APPLICATION TO ASTROMETRY AND PHOTOMETRY

Our first application of the tools derived from information theory to astronomy has been in the area of fundamental bounds to astrometry and photometry, using the Cramér-Rao bound (CR hereafter). The CR bound is an extremely important theoretical result of statistics that indicates that, given a set of measurements that are driven by an underlying distribution function that depends on an (a priori) unknown parameter θ, then the variance of *any* unbiased estimator of θ (no matter *how* you compute the parameter!) will be always larger (or equal) than a minimum (floor) value, which is called the CR bound (a detailed derivation of the CR can be found, e.g., in "Kendalls Advanced Theory of Statistics", Stuart et al. (2004)). The CR bound is built within the framework of parametric statistics, and as such it requires

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[3]See, e.g., `http://en.wikipedia.org/wiki/Information_theory`

specifying a *model* of the observations. Our model involves the simplest case of a linear (1-dimensional) CCD array with a deterministic part (the distribution of light consists of a PSF that characterizes the source, plus a background from the sky and the detector itself), and a stochastic part (the distribution of intensities follows a Poisson distribution driven by the expected flux at each pixel).

Using the above ingredients, it is possible to show, e.g., that for a Gaussian-like PSF, when $\Delta x/FWHM < 1$ (oversampled images, Δx is the detector pixel size), the maximum astrometric precision[4] that one might *ever* get (from an unbiased estimation of x_c, the astrometric position of the source) is given by (Mendez et al. 2014):

- If the detection is dominated by the flux of the source ($B/F \ll 1$):

$$\sigma_{x_c}^2 \approx \frac{1}{8\ln 2} \cdot \frac{1}{GF} \cdot FWHM^2. \qquad (1)$$

- If the detection is dominated by the background ($F/B \ll 1$):

$$\sigma_{x_c}^2 \approx \frac{1}{4\ln 2}\sqrt{\frac{\pi}{2\ln 2}} \cdot \frac{B}{GF^2} \cdot \frac{FWHM^3}{\Delta x}. \qquad (2)$$

In the above equations G is the detector gain (in e$^-$/ADU), B is the background per pixel (in ADUs) and F is the total flux of the source (also in ADUs). If Δx and $FWHM$ are in arcsec, then σ_{x_c} will be also in arcsec.

3. OUTLOOK

It is clear that there are many applications of the CR bound as presented above. Just to name a couple: (1) Observing proposal preparation/observational planning and strategy: What S/N do I require, for a given telescope + detector + observing conditions to reach a certain astrometric precision goal (driven by my scientific goals)?, since big observatories usually provide users with "Exposure Time Calculators", they might also provide "CR Calculators" (of all sorts - not just for astrometry or photometry, e.g., fringe positioning in interferometry) to their users as a general purpose tool, and (2) System performance monitoring: In certain applications, specially space-based, where the environmental conditions are more stable, one might monitor the behavior of the (empirically determined!) CR bound to determine the health of the system. For example, the Gaia experiment regularly checks the CR bound to determine when it is necessary to refocus the telescope (Mora et al. 2014), a very delicate process that should be done only when absolutely necessary to avoid disturbing the astrometric stability of the satellite.

Not all in life is sweet, the CR theorem has two basic shortcomings: It does not tell us how to construct an unbiased estimator that reaches (or approaches) the CR bound, and it does not incorporate any prior information we may have about the parameter. To overcome these deficiencies, one should resort to a Bayesian approach using the "van Trees" inequality (Gill and Levit 1995). The advantage of this is that we can also define a bound, similar to the classical CR bound, but incorporating any prior information from the beginning. Additionally the Bayesian approach comes with an estimator for the parameter built-in: The "conditional expectation", which has theoretical guarantees to reach a minimum mean square error. We are precisely working on these topics right now, and we will probably want to report about them in the next ADeLA meeting.

[4]The same can be done for photometry, lack of space prevents us from showing our results here, please see Mendez et al. (2014).

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