Revista Mexicana de Astronomía y Astrofísica Serie de Conferencias, 60, 64-66, (2026)

OPEN ACCESS https://doi.org/10.22201/ia.14052059p.2026.60.67

"XVII LARIM", Montevideo, Uruguay, November 27 - December 1, 2023.Volume Editors: Gonzalo Tancredi, Cecilia Mateu & Juan José Downes

Orbital resonances in planetary systems (Invited contribution)

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Keywords: *exoplanets, orbital resonances*

Abstract

Orbital resonances are dynamic mechanisms characterized by a strength capable of overcoming other perturbations, managing to maintain the planetary systems captured in the resonant configuration. They are located at some specific values of the semimajor axes but they occupy a non-negligible region of that space. The existence of a certain resonant width added to the fact that there are mechanisms that lead to resonances (through orbital migration for example) makes resonant systems frequent. We show here some applications of the semi-analytical model of Gallardo et al. (2021) which is especially good for eccentric and inclined systems.

Resumen

Las resonancias orbitales son mecanismos dinámicos caracterizados por una fuerza capaz de superar otras perturbaciones, logrando mantener los sistemas planetarios capturados en la configuración resonante. Se ubican en ciertos valores específicos de los semiejes mayores pero ocupan una región no despreciable de ese espacio. La existencia de un determinado ancho resonante sumado a que existen mecanismos que conducen a resonancias (a través de migración orbital por ejemplo) hace que los sistemas resonantes sean frecuentes. Mostramos aquí algunas aplicaciones del modelo semianalítico de Gallardo et al. (2021) que es especialmente bueno para sistemas excéntricos e inclinados.

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1. Mean motion resonances

A stable planetary system has a certain number of fundamental frequencies which are constant and related to the orbital evolution of the planets. High frequencies are given by the orbital periods and low frequencies are defined by the secular evolution of the orbits. Orbital resonances in planetary systems occurs in two flavors: secular resonances and mean motion resonances (MMRs). Secular resonances occur when there are commensurabilities involving the low frequencies. MMRs between planets 1 and 2 occur when there is a commensurability between the planetary orbital periods or mean motions, n. So, we have $k_1n_1 - k_2n_2 \simeq 0$, (e.g. $2n_1 - 3n_2 \simeq 0$) were the k_i are integers. A more sensible indicator of the resonant state is the critical angle $\sigma = k_1\lambda_1 - k_2\lambda_2 + \gamma$ which involves positions (λ) instead of angular velocities (n). When two planets are trapped in MMR their mutual perturbations make their orbits to oscillate (librations) with periods of the order of $\sim 10^2$ orbital periods. Their semimajor axes a_i oscillate and σ also does. MMRs are not an instantaneous effect but a cumulative effect of synchronized planetary perturbations that makes the orbits librate due to a kind of forced mode providing stability against other perturbations. MMRs are very common among planetary systems because planets migrate due to cosmogonic reasons or physical reasons and eventually they fall in resonance. We know the extreme cases of resonant chains with several planets in resonance like TRAPPIST-1 or HD 110067. Several planetary systems exhibits some offset (Charalambous et al., 2022) being the Solar System one of them because the giant planets are close but outside mutual MMRs. Michtchenko & Ferraz-Mello (2001) showed that the Jupiter-Saturn configuration in the exact resonance makes the planetary system chaotic, so the transition to a resonant configuration from one non resonant cannot be done for free. We have also examples of

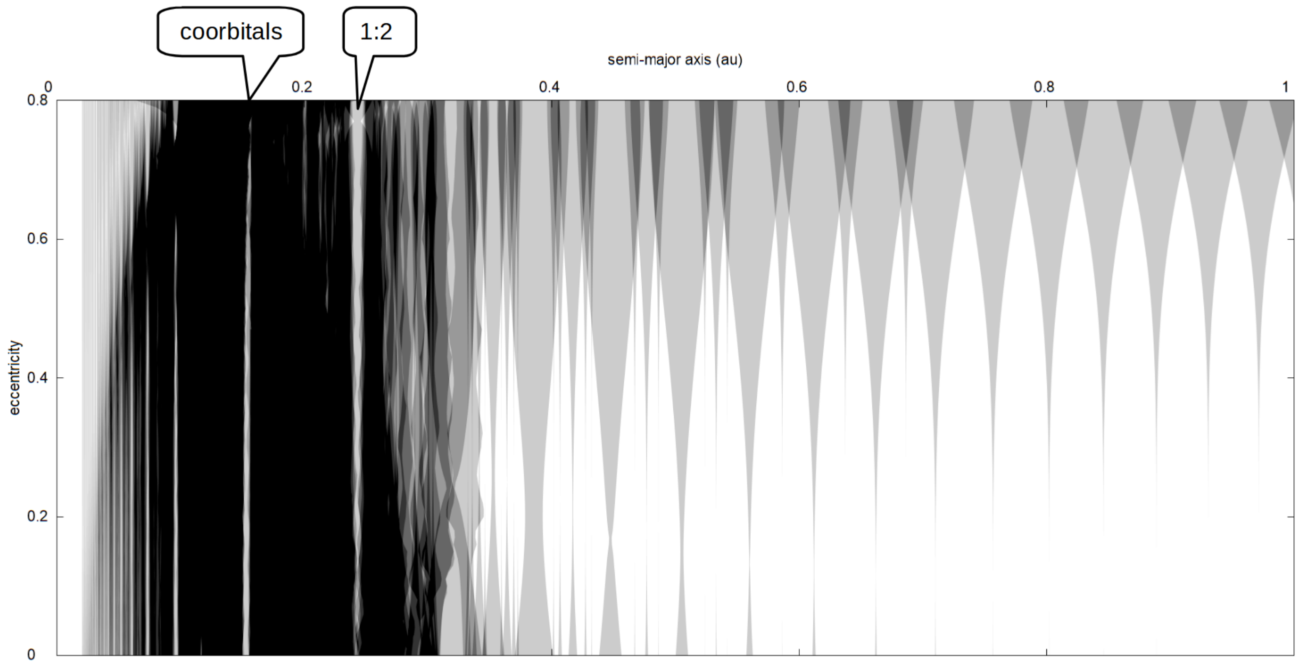

Figure 1. Resonance domain in the space (a, e) of a 1 $m_\oplus$ planet coplanar with the very eccentric planet TOI-3362b. Each resonance domain is painted in grey. Dark regions correspond to resonance overlap. Resonances 1:1 and 1:2 are labeled. The structures dominating the right region correspond to resonances 1:N exterior to TOI-3362b. $\sim$ 300 resonances are showed in this plot.

the contrary situation: Gallardo et al. (2021) showed that the HD 31527 system only can be stable if the two external planets are inside the 16:3 resonance.

There exist MMR models for coplanar and low eccentricities orbits but not models for arbitrary planetary orbits. In order to massively explore the properties of the planetary resonances we proposed a simple model (Gallardo et al., 2021) that works very well for all resonances, arbitrary (e, i), can easily calculate hundreds of resonances and its codes are for non-experts. For example, the model predicts that the libration amplitudes of the planets verify

$$\frac{\Delta a_1}{\Delta a_2} \sim \frac{m_2}{m_1}\left(\frac{k_1}{k_2}\right)^{4/3}$$

that means less massive planets have larger libration amplitude. A recent example of the application of the model to the planetary resonance $1n_1 - 2n_2$ can be found in Giuppone et al. (2023). Codes and materials can be found at https://sites.google.com/view/mmresonances.

2. The case of TOI-3362

According to Dong et al. (2021) this sytem is composed by a star with $M = 1.445M_\odot$ and a planet with $m = 5M_{Jup}$, $a = 0.153$ au, $e = 0.815$.

Applying the model we studied the resonances with an hypothetical 1 Earth mass planet. The results are shown in Figure 1. Each resonance domain in (a, e) of the 1 $m_\oplus$ planet is plotted in grey. We show interior and exteriors resonances. Dark regions correspond to resonance overlap which is an indicator of chaos. There is a strong chaotic region close to the giant planet but with apparently at least two non perturbed regions inside: coorbitals and planets in the 1:2 resonance. For this last case we tested the stability with MEGNO indicator (Cincotta & Simó, 2000), dynamical maps and direct numerical integration of the full equation of motions and confirmed that planets remain stable by at least for $\sim 10^4$ orbital periods. In cases of planetary systems like this one with extreme eccentricities we can predict the properties of mutual resonances very well, but the long term evolution depends crucially on their mutual inclination, which is an undetermined parameter.

3. Conclusions

The model works very well even for high eccentricity cases and when accompanied by dynamical or chaos maps a good dynamical characterization can be obtained. It seems that systems cannot enter or exit a resonance without a dynamic crisis. Why are some resonant systems stable and others unstable? More studies of MMRs with eccentric planets and more secular resonant evolution studies in the spatial case are needed to answer these questions.

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