containing 256^3 grid cells. We assign ionizing luminosities via a constant light-to-(baryonic)mass ratio. This ratio is larger for low mass halos: $\sim 1.5 \times 10^{58}$ ionizing photons per $M_\odot$ for halo masses $< 10^9\ M_\odot$ and $\sim 1 \times 10^{57}$ for those above $10^9\ M_\odot$. This is motivated for example by the different composition in terms of stellar populations and by different escape fractions due to more clumping and more dust in in high mass halos. Additionally, we suppress sources in low mass halos if they are located in regions with an ionization fraction larger 10%, for details see Iliev et al. (2007). The simulation analyzed here is 53Mpc_g8.7_130S in Friedrich et al. (2011).

2. INTRODUCTION TO THE EULER CHARACTERISTIC

The Euler Characteristic (henceforth V_3) is one of four topological invariant Minkowski functionals in three dimensions. For bodies defined by closed surfaces, V_3 can be defined as V_3 = Number of parts-number of tunnels + number of cavities. With this definition, we follow Schmalzing & Buchert (1997) whose algorithm we are using. With this definition, V_3 of a football is 1–0+1 = 2; V_3 of a solid torus is 1–1+0=0 and V_3 of a solid sphere is 1–0+0=1. The Euler Characteristic has been used in many fields of physics, also in cosmology to characterize the topology of large scale structure (e.g. Mecke, Buchert, & Wagner 1994).

2.1. V_3 *of a Gaussian random field (GRF)*

To calculate V_3 of a GRF, we need to divide the continuous field into background (BC) and foreground cells (FC). One can use a threshold value $x_{\rm th}$ to do so: all points with values below $x_{\rm th}$ are BC, the remaining are FC; the FC define the bodies. Choosing a low $x_{\rm th}$ results in many disconnected background objects (holes), choosing a very high threshold value, results in many disconnected bodies. In Figure 1 (left panel) we plot $V_3(x_{\rm th})$ of a GRF. The topology obtained at low $x_{\rm th}$ is called swiss-cheese-topology, the complex topology obtained at intermediate $x_{\rm th}$ is called sponge-topology and the topology at high $x_{\rm th}$ is called meatball-topology.

3. V_3 OF THE SIMULATED IONIZATION FRACTION FIELD DURING EOR

Our goal is to investigate the evolution of the topology of the ionization fraction field in our EoR simulations as a function of redshift, or equivalently, average ionization fraction $\langle x \rangle$. Therefore, we need to choose a $x_{\rm th}$ below which cells count as neutral (BC) and above which cells count as ionized (FC).

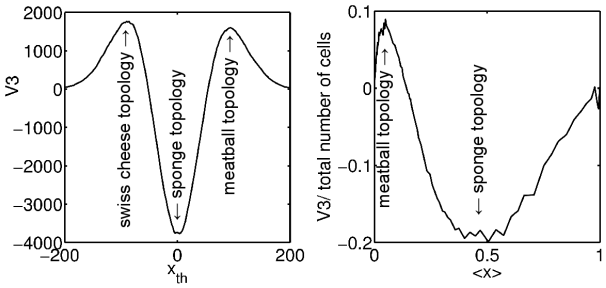

Fig. 1. Left: V_3 of a GRF as a function of $x_{\rm th}$; right: $V_3(\langle x \rangle)$ of the ionization fraction field at $x_{\rm th} = 0.5$. The small fluctuations at high $\langle x \rangle$ are due to our implementation of photons traveling longer distances than a simulation box-length, they are not important here.

In the case of sharp ionization fronts, which are to be expected in this simulation since only black body sources are included, V_3 should be independent of the chosen $x_{\rm th}$. Due to finite resolution, and problems related to edge- and corner- connections of BCs and FCs, there is actually some dependence on $x_{\rm th}$. Between $x_{\rm th} = 0.2$ and 0.6, the evolution of V_3 is largely independent on the actual choice of $x_{\rm th}$; in the right panel of Figure 1 we show $V_3(\langle x \rangle)$ at $x_{\rm th} = 0.5$.

V_3 of the ionization field evolves in the first half as V_3 of a GRF but at high $\langle x \rangle$ there is no rise to positive values followed by a decrease. This shows that if there exists a monotonic steady functional relation between density and time of ionization it does so only until a certain density: low-density areas (the voids in the density field) are ionized "before their time" and do not serve as positive contributions to V_3 in the ionization field. This is a feature of what is called inside-out reionization: the H II regions start in high density regions but do eventually break out in the voids.

The Euler Characteristic can be used to test the effect of different source properties on the topology of the ionization fraction field, see Friedrich et al. (2011).

REFERENCES

Friedrich, M. M., et al. 2011, MNRAS, 413, 1353
Iliev, I. T., Mellema, G., Shapiro, P. R., & Pen, U.-L. 2007, MNRAS, 376, 534
Iliev, I. T., et al. 2008, arXiv:0806.2887v1
Komatsu, E., et al. 2009, ApJS, 180, 330
Mecke, K. R., Buchert, T., & Wagner, H. 1994, A&A, 288, 697
Mellema, G., Iliev, I. T., Alvarez, M. A., & Shapiro, P. R. 2006, NewA, 11, 374
Merz, H., Pen, U.-L., & Trac, H. 2005, NewA, 10, 393
Schmalzing, J., & Buchert, T. 1997, ApJ, 482, L1
Willott, C. J., et al. 2007, AJ, 134, 2435