at different z, while the information provided by the ΛCDM theory allowed to assign halos to galaxies and "individualize" average galaxy $M_{\rm s}$ tracks as a function of mass. By using the inferred $M_{\rm s}$ tracks (GHETs), the SSFRs as a function of $M_{\rm s}$ and z, the evolution of the migration or quenching mass $M_{\rm migr}$, and the rate per unit of comoving volume of galaxies transiting from active to passive ones were predicted. The fact that these predictions are in reasonable agreement with direct observational measures at different z implies that *both the connection between galaxies and ΛCDM halos, and the predicted halo MAHs are consistent with observations.*

The semi-empirical results imply in a natural way the bimodal distribution of galaxies into active (blue) and passive (red) ones, with a bimodality mass scale that shifts downward as time progress (more galaxies become on average passive), this mass scale being at $z = 0$, $M_{\rm s} = 2 - 3 \times 10^{10}\ M_{\odot}$, in agreement with direct observations (e.g., Bell et al. 2003; Kauffmann et al. 2003; Blanton & Moustakas 2009). Therefore, *mass seems to be the main driver of color, SFH, and SF quenching,* though environment is expected to play also a role. Observational studies show indeed that color and SFR are moderately modulated by environment for intermediate-mass galaxies, which assembled more recently, in epochs when larger-scale structures collapse, affecting this those galaxies living in such overdense regions (Cucciati et al. 2010; Peng et al. 2010).

The inference of SSFR (as well as $M_{\rm migr}$) as a function of z was under the hypothesis that the $M_{\rm s}$ growth is driven only by local SF, i.e., the growth by accretion of stellar systems (dry mergers) was excluded. The rough agreement between predictions and direct measurements implies that this hypothesis is reasonable or, said in other words, that the former growth channel dominates.

4. CHALLENGES AND CONCLUDING REMARKS

The semi-empirical results imply that *galaxy formation inside the growing ΛCDM halos is consistent in general with observations.* However, these results show that the average $M_{\rm s}$ assembly histories of galaxies shift from those of their halos in a peculiar way and depending on $M_{\rm h}$ (Figure 3). The complex astrophysical processes and environmental effects involved in galaxy evolution should explain this. Among the key challenges, we highlight the following ones:

4.1. *Why the $M_{\rm s}$ assembly of low-mass galaxies is systematically delayed with respect to the assembly of their halos?*

The SAMs introduce efficient SN-driven galaxy outflows for reproducing the shallow faint-end of the luminosity function or, equivalently, the high inclination of $M_{\rm s}/M_{\rm h}$ at lower masses (Figure 2). Can galaxy outflows explain also the inefficient galaxy SF in the past as smaller are the halos? By means of self-consistent disk-galaxy evolutionary models, Firmani, Avila-Reese & Rodríguez (2010) have shown that SN-driven outflows, as well as the local SF and ISM feedback processes, may deviate the SFR history of disk galaxies from the associated halo mass aggregation rate history but not enough as to reproduce the semi-empirical and empirical results. In Figures 2, 3, and 4 two evolutionary models of low-mass disk galaxies are plotted (dotted-line curves). The model tracks differ significantly from the GHETs. As seen in Figure 3, the model $M_{\rm s}$ tracks follow closely those of their halos at late epochs, in disagreement with the systematical shift as the mass is smaller observed for the corresponding GHETs. Although the low $M_{\rm s}/M_{\rm h}$ ratios were obtained in the models at $z = 0$ (by assuming very high SN kinetic-energy injection to the outflow, see also e.g., Dutton & van den Bosch 2009), at higher z, model prediction are far from the semi-empirical inferences (Figure 2). The disagreement between models and observations is also evident by comparing the SSFRs: the former predict SSFRs for small galaxies much lower than the observed ones, specially toward lower z.

Firmani et al. (2010) explored the possibility of later re-accretion of the ejected gas (e.g., Oppenheimer et al. 2010). For reasonable schemes of gas re-accretion as a function of $M_{\rm h}$, they found that the SSFR of galaxies increases but in the opposite direction of the downsizing trend: the increase is large for massive galaxies and small for the less massive ones. From the side of SAMs, it was found that the population of small galaxies (both central and satellites) is too old, red, and passive as compared with observations (e.g., Somerville et al. 2008; Fontanot et al. 2009; Santini et al. 2009; Liu et al. 2010), issues that certainly are related to the mentioned-above problem of the $M_{\rm s}$ buildup of ΛCDM-based sub-M^* model galaxies. N-body + hydrodynamics cosmological simulations of central low-mass galaxies ($0.2 \lesssim M_{\rm s}/10^9 M_{\odot} \lesssim 30$ at $z = 0$) show similar problems (Colín et al. 2010; Avila-Reese et al. 2011): (1) the SSFRs are 5–10 times lower than the average of observational determinations at $z \sim 0$, an inconsistency that apparently re-