in finding a solution $\tau = \tau(t)$

$$\dot{\Omega} = -k\Omega^n \rightarrow \tau = \frac{\tau_0}{f(t)} + \frac{k_0}{k}\int_0^t f(t)dt\,, \quad (3)$$

where τ_0 is the initial spin-down age. The braking index can indeed be measured from the second derivative of the period (presently only available for a few radio pulsars) and have a general formulation too

$$n = n_* + \frac{\dot{f(t)}}{f(t)}\frac{P}{\dot{P}}\,, \quad \text{with} \quad n_* = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 2 - \frac{P\ddot{P}}{\dot{P}^2}\,. \quad (4)$$

Finally, it is possible to obtain a general expression for the temporal evolution of the spin-down luminosity $\dot{E} = I\Omega\dot{\Omega}$ as

$$\dot{E} = \dot{E}_0 f(t)\left(\frac{\tau}{\tau_0}f(t)\right)^{-\frac{n+1}{n-1}}\,, \quad (5)$$

where $\dot{E}_0$ is the initial spin-down luminosity.

When k is constant we recover the standar result for the spin-down age, braking index and spin-down luminosity.

2.1. *Case* $k = k_0 f(t)$

The more reasonable funtion $f(t)$ that satisfies $f(0) = 1$ and the condition of growth of $\mathbf{B}$ is $f(t) = \varepsilon + (1 - \exp(-\frac{t}{\tau_B}))$.

In this case we assume $\varepsilon \ll 1$ and τ_B is the field growth time-scale. The solution to spin-down age, the braking index and the spin-down luminosity for this funtion are

$$\tau = \frac{\tau_0 + t}{f(t)} - \tau_B\,, \quad (6)$$

$$n = n_* + \frac{\exp(-t/\tau_B)}{\tau_B f(t)}\frac{P}{\dot{P}}\,, \quad (7)$$

$$\dot{E} = \dot{E}_0 f(t)\left(1 + \frac{t}{\tau_0} - f(t)\frac{\tau_B}{\tau_0}\right)^{-\frac{n+1}{n-1}}\,. \quad (8)$$

The Figure 1 shows the Spin-down luminosity for several values of τ.

2.2. *Energy Change*

In addition, we want to find a general relation for the energy change based on the observable parameters. In this case:

$$E = \frac{1}{2}I\Omega^2 = \frac{(n-1)}{2}\tau\dot{E}\,, \quad (9)$$

$$\tau = \frac{1}{(n-1)}\frac{P}{\dot{P}}\,, \quad (10)$$

$$\dot{E} = I\Omega\dot{\Omega} = -4\pi^2 I P^{-3}\dot{P}\,, \quad (11)$$

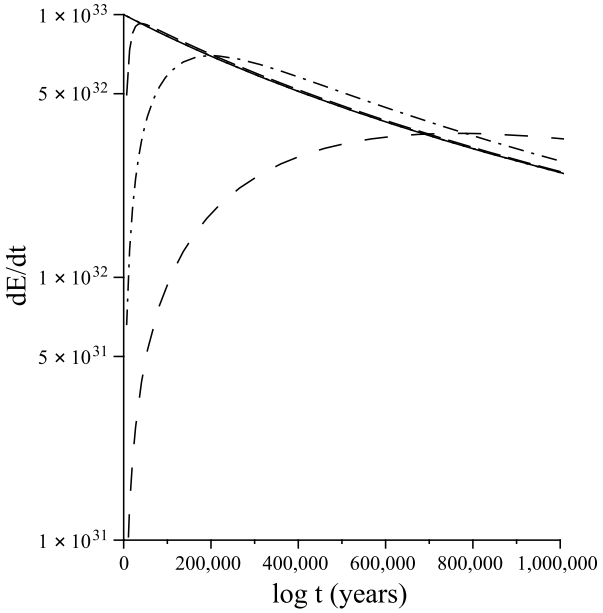

Fig. 1. Spin-down luminosity varying the spin-down initial age.

and we obtain

$$\Delta E = E_0 - E = \frac{(n-1)}{2}\left(\tau_0\dot{E}_0 - \tau\dot{E}\right)\,.$$

For the magnetic field growing case, the initial spin-down, the initial luminosity and the energy change allow us to associate the observable parameters with the theoretical model

$$\tau_0 = (\tau + \tau_B)f(t) - t\,, \quad (12)$$

$$\dot{E}_0 = \frac{\dot{E}}{f(t)}\left(\frac{\tau f(t)}{\tau_0}\right)^{\frac{n+1}{n-1}}\,, \quad (13)$$

$$\Delta E = \frac{(n-1)}{2}\dot{E}\left[\left(\frac{\tau f(t)}{\tau_0}\right)^{\frac{n+1}{n-1}}\left(\frac{\tau_0}{f(t)}\right) - \tau\right]\,.$$

For the SN1987A case, a CCO can emit less than the observed limits from this supernova even if 100 per cent of its spin-down power is reprocessed into IR emission by dust in the surrounding SN ejecta. So, we conclude that a CCO is a promising model for an unseen NS in SN1987A.

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