One fits a quadric surface with ten coefficients, $a, b, c, d, e, f, g, h, k, l$, to these velocities. To assure that the equation indeed corresponds to an ellipsoid one must impose the condition that the matrix of the a, b, c coefficients

$$A = \begin{pmatrix} a & c/2 & d/2 \\ c/2 & b & e/2 \\ d/2 & e/2 & c \end{pmatrix},$$

in fact be positive-definite and symmetric. To avoid the trivial solution $a = b = \cdots = q = 0$ another condition must be imposed. The one I use is that the volume of the ellipsoid must be a maximum. Because the volume is proportional to the determinant of A, the condition becomes $\det(A) = \max$. An eigenvalue-eigenvector decomposition of the matrix A yields the axes of the velocity ellipsoid and their orientation with respect to the Galactic coordinate system. Because of generally greater error in the data, the coefficients $a, \ldots, l$ are calculated by use of the robust L_1 criterion. SDP allows one to combine without difficulty the least squares criterion for the kinematical parameters and the L_1 criterion for the velocity ellipsoid. For the latter if there are n space velocities with n corresponding residuals $r_{sv,i}$, let $\gamma_i, i = 1, \ldots, n$, be n *positive* parameters. Then minimize the γ in the objective function subject to the conditions

$$\mathrm{diag}(\gamma + r_{sv}) > 0; \qquad \mathrm{diag}(\gamma - r_{sv}) > 0; \qquad \mathrm{diag}(\gamma) > 0.$$

The SDP formulation of the velocity ellipsoid calculation along with the L_1 criterion for the minimization of the residuals offers advantages over competing methods. The ellipsoid calculated is unique and represents a global minimum of the reduction criterion when that criterion is the L_1 (Calafiore 2002). Bochanski, Hawley, & West (2011) prefer a geometric simplex minimization with the least squares criterion. Although simplex minimization can be used with the L_1 criterion, thus making the procedure robust-see the algorithm in Branham (1990, pp. 191–197), the method can converge to a local rather than to a global minimum, particularly when many parameters are being fit, unless a good starting approximation is available. Pourbaix (1998) , using the simplex algorithm to calculate orbits of double stars, estimates that when n parameters are being fit there are $\approx \mathrm{O}(e^n)$ local minima. He implements a simulated annealing modification of the simplex algorithm to reject the local minima. With SDP and the L_1 criterion such a strategy becomes unnecessary because the minimum *is* global.

The solar velocity, $S_0 = \sqrt{X^2 + Y^2 + Z^2}$, calculated from both the solution for the kinematical parameters $X, Y, \ldots, w_y, w_z$ and the coefficients of the velocity ellipsoid must be the same. This condition can be imposed when one uses the SDP formulation of the problem.

5. SOME CORRECTIONS TO THE OBSERVATIONS AND COVARIANCE MATRICES

The total space motions needed in the velocity ellipsoid calculation should be corrected for the effects of Galactic rotation by modifying the proper motions and radial velocities used in the calculations to remove the rotation. This was done by the same procedure used in Branham (2009a).

In theory one should also apply a correction for the incompleteness of the sample of the G giant stars taken from the Hipparcos catalogue. Trumpler & Weaver (1962, p. 374) define a factor of incompleteness K_1 as

$$K_1 = \frac{N(m, \mu)}{N_\varpi(m, \mu)}, \tag{4}$$

where $N(m, \mu)$ is the number of stars in the sky for magnitude interval $m \pm \Delta m/2$ and proper motion interval $\mu \pm \Delta\mu/2$ and $N_\varpi(m, \mu)$ is the number of stars in the parallax catalogue for the same intervals. Equation (4) is difficult to apply if there is insufficient overlap between the proper motion catalog and the parallax catalog. For the Hipparcos parallaxes a logical proper motion catalog would be the Tycho II catalog (Høg et al. 2000). But for K_1 the sparse overlap between the two catalogs assures that the factor becomes large with large mean errors. One must question whether such corrections are realistic and should be applied. I feel they should not and that the randomness of the data is more important.

Trumpler & Weaver (1962, p. 375) also define a second incompleteness factor, K_2, to correct for the absence of proper motions in the parallax catalog nearly along the line of sight and hence undetectable. K_2 depends