sponding on average to a given $M_{\rm s}$ is taken from the empirical blue and red $M_{\rm g}$–$M_{\rm s}$ relations. In Figure 2 a compilation of observational estimates is plotted in the $M_{\rm s}$–$M_{\rm g}$ plane for a sample of disk galaxies that includes low surface brightness galaxies from Avila-Reese et al. (2008; blue dots with error bars; they added H_2 mass contribution by using an estimate for the H_2-to-HI mass ratio as a function of galaxy type), and for another galaxy sample from McGaugh (2005; blue crosses; no H_2 contribution is considered and their dwarf galaxies were excluded). An orthogonal linear doubly-weighted regression to the data from Avila-Reese et al. (2008) gives:

$$\frac{M_{\rm g}}{10^{10}M_{\odot}} = 0.43 \times \left(\frac{M_{\rm s}}{10^{10}M_{\odot}}\right)^{0.62}. \qquad (2)$$

This fit is plotted in Figure 2 with its corresponding estimated scatter (≈0.3 dex; blue solid and dashed lines). This is the relation and its scatter used to calculate $M_{\rm b}$ and the blue *GBMF*. A similar relation has been inferred by Stewart et al. (2009). The gas fractions in red galaxies are much smaller than in blue galaxies. For sub-samples of blue and red galaxies, Wei et al. (2010) reported for each one the atomic gas fractions versus $M_{\rm s}$ (molecular gas was not included). The ratio of their fits to these data as a function of $M_{\rm s}$ is used here to estimate from equation 2 (blue galaxies) the corresponding average $M_{\rm g}$ for red galaxies as a function of $M_{\rm s}$. The red solid line shows the obtained relationship. As an approximation to the scatter (short-dashed lines), the average scatter reported for red galaxies in Wei et al. (2010) is adopted here.

2.2. *Halo and sub-halo mass functions*

A great effort has been made in the last decade to determine the *HMF* at $z = 0$ and at higher redshifts by means of N-body cosmological simulations. A good fit to the results, at least for low redshifts, is the universal function derived from a Press-Schechter formalism (Press & Schechter 1974) generalized to the elliptical gravitational collapse (Sheth & Tormen 1999, hereafter S-T). In fact, Tinker et al. (2008) have shown that at a high precision level, the *HMF* may change for different cosmological models and halo mass definitions as well as a function of z. For our purposes and for the cosmology used here, the S-T approximation provides a good description of the $z = 0$ *HMF* of distinct halos:

$$\phi_h(M_h)dM_h = A\left(1+\frac{1}{\nu^{2q}}\right)\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}_M\nu}{M_h^2}\left|\frac{d\ln\sigma}{d\ln M_h}\right|\exp\left[-\frac{\nu^2}{2}\right]dM_h, \qquad (3)$$

where $A = 0.322$, $q = 0.3$, $\nu^2 = a(\delta_c/D(z)\sigma(M_{\rm h}))$ with $a = 0.707$; $\delta_c = 1.686\Omega_m^{0.0055}$ is the linear threshold in the case for spherical collapse in a flat universe with cosmological constant, $D(z)$ is the growth factor and $\sigma(M_{\rm h})$ is the mass power spectrum variance of fluctuations linearly extrapolated to $z = 0$. The halo (virial) mass, $M_{\rm h}$ is defined in this paper as the mass enclosed within the radius where the overdensity is $\bar{\rho}_{\rm vir} = \Delta$ times the *mean matter* density, $\bar{\rho}_M$; $\Delta \approx 340$ according to the spherical collapse model for the cosmology used here. The cosmological parameters assumed here are close to those of WMAP5 (Komatsu et al. 2009): $\Omega_M = 0.27, \Omega_\Lambda = 1-\Omega_m = 0.73, h = 0.70, \sigma_8 = 0.8$.

The distinct *HMF* should be corrected when a *GSMF* corresponding to *all* galaxies is used in the AMT. In this case, satellite galaxies are included in the *GSMF*. Therefore, subhalos should be taken into account in the *HMF*. The subhalo fraction is no more than ≈20% of all the halos at $z = 0$ (e.g., Shankar et al. 2006; Conroy et al. 2006; Giocoli et al. 2010; BCW10). When necessary, we correct the S-T *HMF* for (present-day) subhalo population by using the fitting formula to numerical results given in Giocoli et al. (2010):

$$\frac{dn(m_{\rm sub})}{d\ln m_{\rm sub}} = A_0 m_{\rm sub}^{\eta-1}\exp\left[-\left(\frac{m_{\rm sub}}{m_0}\right)^{\gamma}\right], \qquad (4)$$

with $\eta = 0.07930$, $\log A_0 = 7.812$, $\log(m_0/M_{\odot}) = 13.10$ and $\gamma = 0.407$.

The upper panel of Figure 3 shows the (distinct) S-T *HMF* (solid line), the sub-halo *HMF* (short-long-dashed line), and the distinct+subhalo *HMF* (dash-dotted line). The correction by sub-halos in the abundance is small at low masses and negligible at high masses. When the *GSMF* refers only to central galaxies –which is the case in this paper– then it is adequate to use the distinct *HMF* for the AMT, i.e. *the subhalo abundance correction is not necessary.*

2.2.1. *Halos hosting blue and red galaxies*

In the AMT, the cumulative *GSMF* and *HMF* are matched in order to link a given $M_{\rm s}$ to $M_{\rm h}$. When a subsample of the total *GSMF* is used –as is the case for inferring the $M_{\rm s}$–$M_{\rm h}$ relation of only late- or early-type galaxies– it would not be correct to use the total *HMF* for the matching. This function, in the ignorance of which is the mass function of halos hosting blue (red) galaxies, at least should be re-normalised (decreased uniformly) by the same fraction corresponding to the decrease of the subsample *GSMF* with respect to the total *GSMF*. In