outflows. Such an analysis was not possible before because the HH 34 and 111 jets, counterjets, and outflow sources were not simultaneously detected in previous optical and/or near-IR images. Therefore, the *Spitzer* images open up a new possible analysis of HH outflows, because even though they have somewhat lower angular resolution than previous optical/IR images, they simultaneously show the source and the knots along the two outflow lobes.

Motivated by these new observations, in the present paper we develop a model of a jet with quasi-symmetric ejections of ballistic knots. The model predicts the behavior of the spatial offsets between jet/counterjet knot pairs as a function of increasing distance from the outflow source. The predicted dependence can then be compared with the knot positions measured in observed HH outflows in order to constrain the model parameters. A simpler model of this kind was described by Raga et al. (2011).

The paper is organized as follows. The ballistic, binary ejection model is described in § 2 (both the non-relativistic and the relativistic cases are considered). An application of the non-relativistic model to the observed positions of the jet/counterjet knots in HH 34 and 111 is described in § 3. Finally, the implications of the results are discussed in § 4.

2. A BALLISTIC, QUASI-SYMMETRIC BINARY EJECTION MODEL

2.1. *Non-relativistic flows*

Let us consider a non-relativistic, bipolar ejection with imperfect symmetry. In one direction (along the "jet"), a clump is ejected at a time τ with a velocity v_j (projected on the plane of the sky). In the opposite direction (along the "counterjet"), a clump is ejected at a time $\tau+\Delta\tau$, with a velocity $v_j-\Delta v$. If the parcels are ballistic, at a later time t they will be at distances x_j and x_{cj} (on the plane of the sky, along the jet and the counterjet, respectively) given by:

$$x_j = (t-\tau)v_j\,; \quad x_{cj} = (t-\tau-\Delta\tau)(v_j-\Delta v)\,. \tag{1}$$

From this equation, one obtains that the offset $\Delta x = x_j - x_{cj}$ (between the positions of the jet and counterjet knots) grows with distance from the source, following

$$\Delta x = x_j - x_{cj} = \Delta\tau v_j + \frac{\Delta v}{v_j} x_j\,, \tag{2}$$

where we have neglected the term $\Delta\tau\,\Delta v$ (which involves the product of the two perturbations).

Now, let us assume that we have an ensemble of knot pairs, with $\Delta\tau$ and Δv values uniformly distributed in intervals $[\tau_0-\Delta\tau_0, \tau_0+\Delta\tau_0]$ and $[v_0-\Delta v_0, v_0+\Delta v_0]$, respectively. In other words, we allow for an intrinsic asymmetry in the ejection velocity Δv and in the time-delay $\Delta\tau$ between the jet/counterjet ejections.

The ensemble average of equation (2) is:

$$<\Delta x> = \int_{v_0-\Delta v_0}^{v_0+\Delta v_0}\int_{\tau_0-\Delta\tau_0}^{\tau_0+\Delta\tau_0} \Delta x\;\frac{d\Delta\tau\,d\Delta v}{4\Delta\tau_0\Delta v_0}$$
$$= \tau_0 v_j + \left(\frac{v_0}{v_j}\right) x_j\,. \tag{3}$$

Therefore, the ensemble average of Δx as a function of x_j (where x_j is the distance to the successive knots measured along the jet) is a straight line with intercept a_1 and slope b_1 such that:

$$a_1 = \tau_0 v_j\,; \qquad b_1 = \frac{v_0}{v_j}\,. \tag{4}$$

We should note that if we have an error of magnitude Δx_s in the estimated position of the source, we will obtain $a_1 = \tau_0 v_j \pm \Delta x_s$.

Also of interest is the quadratic average:

$$<(\Delta x)^2> = \int_{v_0-\Delta v_0}^{v_0+\Delta v_0}\int_{\tau_0-\Delta\tau_0}^{\tau_0+\Delta\tau_0}\left[(\Delta\tau)^2 v_j^2 + \left(\frac{\Delta v x_j}{v_j}\right)^2 + 2\Delta\tau\Delta v x_j\right]\frac{d\Delta\tau\,d\Delta v}{4\Delta\tau_0\Delta v_0}$$
$$= v_j^2\left(\tau_0^2+\frac{\Delta\tau_0^2}{3}\right) + \left(\frac{x_j}{v_j}\right)^2\left(v_0^2+\frac{\Delta v_0^2}{3}\right) + \tau_0 v_0 x_j\,. \tag{5}$$

Therefore, the ensemble average of $(\Delta x)^2$ has a quadratic, $<(\Delta x)^2> = a_2 + b_2 x_j + c_2 x_j^2$ dependence with:

$$a_2 = v_j^2\left(\tau_0^2+\frac{\Delta\tau_0^2}{3}\right)\,; \quad c_2 = \left(\frac{v_0}{v_j}\right)^2 + \frac{1}{3}\left(\frac{\Delta v_0}{v_j}\right)^2\,, \tag{6}$$

and with $b_2 = a_1 b_1$ given by the results of the fit to the $<\Delta x>$ vs. x_j dependence (see equations 4 and 5).

To summarize, if one observes an outflow in which pairs of jet/counterjet knots can be identified, one has to make fits to the Δx vs. x_j and $(\Delta x)^2$ vs. x_j dependencies (where $\Delta x = x_j - x_{cj}$, the observed positional asymmetry between the jet/counterjet knot pairs). If one has an independent estimate of the spatial velocity v_j of the knots, the coefficients of the fits (see equations 4 and 6) directly give us the mean and half width of the ejection time (τ_0 and $\Delta\tau_0$) and velocity (v_0 and Δv_0) distributions.