2.2. *Relativistic flows*

Let us now consider the problem of quasi-symmetric, binary ejections at relativistic velocities. In § 2.1 (in which we considered a non-relativistic flow), we did not introduce the effect of projection onto the plane of the sky because the distances from the source and the jet velocity have the same plane-of-the-sky projection (and therefore, one can work with the projected distances and velocities).

In the case of a relativistic binary ejection, the clump ejected along the jet at a time τ with an intrinsic velocity v_j will be at a time t at a projected distance (from the source):

$$x_j = \frac{(t-\tau)v_j \cos\phi}{1 - v_j \sin(\phi/c)}\,, \tag{7}$$

where ϕ is the angle (towards the observer) between the jet axis and the plane of the sky, and c is the speed of light.

The clump ejected along the counterjet at a time $t + \Delta\tau$ with a velocity $v_j - \Delta v$ will be at a time t at a projected distance:

$$x_{cj} = \frac{(t-\tau-\Delta\tau)(v_j-\Delta v)\cos\phi}{1 + (v_j - \Delta v)\sin(\phi/c)}\,. \tag{8}$$

Combining equations (7-8) and expanding in a Taylor series up to first order in $\Delta\tau$ and Δv we obtain:

$$\Delta x = x_j - x_{cj} = \frac{\Delta\tau v_j \cos\phi}{1 + v_j \sin(\phi/c)}$$

$$+\left[\frac{2v_j \sin(\phi/c)}{(1 + v_j \sin(\phi/c))} + \frac{(1 - v_j \sin(\phi/c))(\Delta v/v_j)}{(1 + v_j \sin(\phi/c))^2}\right] x_j\,. \tag{9}$$

Carrying out the ensemble averages described in § 2.1, we then derive:

$$<\Delta x> = A_1 + B_1 x_j\,;\;\; <(\Delta x)^2> = A_2 + B_2 x_j + C_2 x_j^2\,, \tag{10}$$

with

$$A_1 = \tau_0 v_j A\,, \qquad B_1 = B + C\frac{v_0}{v_j}\,,$$

$$A_2 = A^2\left(\tau_0^2 + \frac{\Delta\tau_0^2}{3}\right)v_j^2\,,\;\; B_2 = 2A(Bv_j + Cv_0)\tau_0\,,$$

$$C_2 = \frac{C^2}{v_j^2}\left(v_0^2 + \frac{\Delta v_0^2}{3}\right) + B^2 + 2BC\frac{v_0}{v_j}\,, \tag{11}$$

where A, B and C are the dimensionless functions

$$A = \frac{\cos\phi}{1 + v_j \sin(\phi/c)}\,, \quad B = \frac{2v_j \sin(\phi/c)}{(1 + v_j \sin(\phi/c))}\,,$$

$$C = \frac{1 - v_j \sin(\phi/c)}{(1 + v_j \sin(\phi/c))^2}\,. \tag{12}$$

A, B and C are functions of order unity (except when $1 + v_j \sin(\phi/c) \ll 1$) for relativistic jets, with $v_j \sim c$. For $v_j \ll c$, they have limits $A = C = 1$ and $B = 0$. Inserting these values in equation (11) it is clear that we recover the non-relativistic equations (3–5) (considering that the velocities used in equation 11 correspond to the full spatial motion and not the projection on the plane of the sky considered in equations 3–5).

3. AN APPLICATION OF THE MODEL TO TWO HH JETS

3.1. *Results of the model fit*

We now apply the non-relativistic ballistic ejection model described in § 2.1 to two HH jets: HH 34 and HH 111. We first calculate least squares fits of the jet/counterjet knot offsets $\Delta x = x_j - x_{cj}$ and of $(\Delta x)^2$ as a function of distance x_j from the source. We then use the coefficients from the fits (together with an estimate of the knot velocities v_j) to constrain the ejection time ($\tau_0 \pm \Delta\tau_0$) and ejection velocity ($v_0 \pm \Delta v_0$) distributions resulting in the observed jet/counterjet asymmetries.

Given the fact that the distances and velocities along the jet/counterjet have identical projections onto the plane of the sky, we carry out the fitting procedure using the angular distances and proper motion velocities measured for the knot pairs. To obtain physical values from the angular distances and velocities, we assume a distance of 417 pc to HH 34 and 111 (see Menten et al. 2007).

3.2. *The observations*

The observations of HH 34 and HH 111 were downloaded from the *Spitzer* Heritage Archive[5]. They are part of our original General Observer (GO) program 3315 (PI Noriega-Crespo) obtained with the infared camera IRAC (Fazio et al. 2004) and the infrared photometer MIPS (Rieke et al. 2004) in March 28, 2005. The high quality of the archival images (Post Basic Calibrated Data or Post-BCD; S18.7 products) was enough for our purposes and no further processing was required. The HH 34 and HH 111 observations with IRAC were taken in its four channels (1, 2, 3, 4) = (3.6, 4.5, 5.8 & 8.0 μm) and with a total integration time per pixel of 360 sec, using 30 sec High Dynamic Range (HDR) exposures. The final images are sampled with 0.6″ per pixel,

[5] `http://sha.ipac.caltech.edu/applications/Spitzer/SHA`.