TABLE 4

PARAMETERS FOR METHOD 2

Distance	Dispersion	Age
Maciel	σ_U	t_9
Maciel	σ_V	t_{10}
Maciel	σ_W	t_{11}
Maciel	σ_T	t_{12}
Stanghellini	σ_U	t_{13}
Stanghellini	σ_V	t_{14}
Stanghellini	σ_W	t_{15}
Stanghellini	σ_T	t_{16}

results. Considering further the equatorial coordinates (α, δ) of the PN, we have used the equations by Boesgaard & Tripicco (1986) to derive the U, V, W velocity components of the nebulae, as well as the total velocity T and the velocity dispersions σ_U, σ_V, σ_W, and σ_T. According to these equations we derive the following parameters: $C = f(d)$, $X = f(C, \mu_\alpha, \mu_\delta, \alpha, \delta, V_r)$, $Y = f(C, \mu_\alpha, \mu_\delta, \alpha, \delta, V_r)$, and $Z = f(C, \mu_\delta, \delta, V_r)$, from which the velocities can be written as $U = f(X, Y, Z)$, $V = f(X, Y, Z)$, $W = f(X, Y, Z)$, and $T = f(X, Y, Z)$, so that the dispersions are given by

$$\sigma_i = \sqrt{(V_i - \bar{V}_i)^2}\,, \tag{5}$$

where V_i stands for the velocities U, V, W, T. Then, we have again used the detailed correlations between the velocity dispersions and the ages as given by the Geneva-Copenhagen survey (Holmberg et al. 2009), adopting the same coefficients given in Table 2. We have used the same distance scales (Maciel 1984; Stanghellini et al. 2008), so that we have again 8 different age distributions, corresponding to the timescales t_9 to t_{16}, as described in Table 4.

In practice, we have considered several additional cases, in order to better investigate the hypothesis of $V_t \simeq V_r$. Assuming that these velocities are of the same magnitude, but allowing for the possibility of different signs, we have as a result several possibilities for the proper motions μ_α and μ_δ, all of which are consistent with either $V_t \simeq V_r$ or $|V_t| \simeq |V_r|$. It turns out that these possibilities produce very similar age distributions, which will be discussed in § 4. Therefore, we will present only the distributions of the ages t_9 to t_{16}, as defined in Table 4, for the cases where $\mu_\alpha \simeq \mu_\delta \simeq 0$.

An interesting alternative to overcome the lack of proper motion and tangential velocity measurements would be to apply the singular value decomposition (SVD) technique, as used by Branham (2010) to solve the inverse problem, that is, obtaining the space velocities from available proper motions. However, in view of the similarity of the results for different assumptions regarding the tangential velocities, it is unlikely that this technique would produce very different results than presented here.

3. THE SAMPLES

As mentioned in the Introduction, we have considered two samples of Milky Way PN. In order to make comparisons with our previous work, we have first considered the same sample used by Maciel et al. (2003, 2005, 2006), which we will call Sample 1. This sample contains 234 well-observed nebulae located in the solar neighborhood and in the disk, for which all data were obtained with the highest accuracy. Their Galactocentric distances are in the range $4 < R(\mathrm{kpc}) < 14$, and most (69%) are located in the solar neighborhood, with distances $d < 3$ kpc.

The second sample considered in this work, called Sample 2, includes all the nebulae for which accurate radial velocities are available in the catalogue by Durand et al. (1998), comprising 867 objects. This is a more complete sample, so that it is expected that the derived results can be extended to the observed population of PN in the Galaxy. In both samples, the number of nebulae used depends on the availability of the statistical distances. The actual numbers of objects from the Maciel (1984) and Stanghellini et al. (2008) distance scales are 195 and 170 for Sample 1 and 493 and 403 for Sample 2, respectively. We have then applied the approximation given by equation (4) for both samples, with the coefficients shown in Table 2, considering only the objects for which ages in the interval $0 < t(\mathrm{Gyr}) < 14$ could be obtained.

4. RESULTS AND DISCUSSION

The main results for the age distribution of the CSPN are shown in Figures 1–4, where we have used the age parameter definitions given in Tables 3 and 4 for Methods 1 and 2, respectively. Figures 1 and 2 refer to Sample 1, while Figures 3 and 4 refer to Sample 2. It can be seen that the age distributions obtained by both methods are similar, in the sense that most objects have ages under 5 Gyr, with a strong peak at ages typically between 1 and 3 Gyr. The histograms of Figures 3–4 are summarized in Table 5, where the fraction of stars obtained by Method 1 (ages t_1 to t_8) and Method 2 (ages t_9 to t_{16}) are shown for three age bins, namely 0-3 Gyr, 3-6 Gyr, and $t > 6$ Gyr.