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THE IMPORTANCE OF NUCLEUS ROTATION IN DETERMINING THE LARGEST GRAINS EJECTED FROM COMETS

A. Molina[1,2]

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RESUMEN

Se determina el valor máximo del diámetro de las partículas eyectadas de los núcleos cometarios haciendo uso del modelo clásico de expansión radial de los gases de los cometas. Se destaca la importancia de las fuerzas inerciales, principalmente de las debidas a la rotación del cometa. Se muestra una sencilla expresión indicando el valor mínimo que debe tener el periodo de rotación del cometa para que la partícula pueda levantarse de la superficie. Los resultados obtenidos son comparados con los valores determinados a partir de medidas de radar. También se analiza el cometa Churyumov-Gerasimenko, que alcanzará la misión ROSETTA en 2014, proponiendo valores para los diámetros de las mayores partículas que pueden desprenderse del cometa.

ABSTRACT

The maximum diameter of large boulders ejected from the cometary nuclei is investigated using the classical model of dust grain as dragged out by radially expanding cometary gases. The importance of the inertial forces, and particularly those due to the rotation of the comet is shown. Although a larger dust grain can be lifted from the nucleus surface if the rotation is faster, the rotation period has to be larger than a certain value. A simple expression for this critical value of rotation period is given. Our results are applied to different comets and a comparison with the maximum radius values obtained by radar measurements is made. Finally, comet Churyumov-Gerasimenko, the target of the ROSETTA mission to arrive on 2014, is going to be analyzed and a range for the maximum diameter of the dust grains that can be lifted is proposed.

Key Words: comets: general — comets: individual (Halley, Wirtanen, Hyakutake, C/2001 A2 LINEAR, IRAS-Araki-Alcock, Churyumov-Gerasimenko) — interplanetary medium — meteoroids

1. INTRODUCTION

As is known, the cometary nuclei release dust grains as the comets approach the Sun. Small grains are dominated by solar radiation pressure and leave the nucleus to form the dust tail of the comet. Large grains can produce a meteroid stream. The study of meteoroid streams can be used as a probe of cometary structure (Beech 1998). Thus, knowledge of the characteristics of meteoroids can provide us information on the composition of cometary nuclei. The large grains can be up to a few meter in diameter. Taking into account the conditions to produce electrophonic sounds, Beech (1998) obtained a lower bound of 1.24 m for the diameter of the largest meteoroids within the Leonid stream. Cometary dust trails have been observed using space-based telescopes, as in Sykes et al. (1986) who used four broad-band filters at 12, 25, 60 and 100 μm on the Infrared Astronomical Satellite. Reach, Kelley, & Sykes (2007) observed debris trails of 27 short period comets due to mm-sized or larger particles using the 24 μm camera on the *Spitzer Space Telescope*. Also, meteoroid strems have been observed by ground-based telescopes. Thus, Ishiguro et al. (2002) obtained first evidence of a cometary dust trail in optical wavelengths using the 2K CCD camera attached

[1]Departamento de Física Aplicada, Facultad de Ciencias, Universidad de Granada, Granada, Spain.

[2]Instituto de Astrofísica de Andalucía, CSIC, Granada, Spain.

to the 105 cm Schmidt telescope at the Kiso Observatory. Aside from the direct use of telescopes, these large meteoroids can be studied by means of the observations of the flashes that occur when they impact on the surface of the Moon. Ortiz et al. (2000) analyzed five impact flashes observed on the night side of the Moon on 18 November 1999, which were associated to the flux of meteoroids of the Leonid meteor shower. They derived a mass range of 1.3 kg to 9 kg for the meteoroid that produced the brightest impact. Assuming that the Leonids are the meteoroids with the lowest density values, 400 kg m^{-3}, of the all known meteoroid streams (Babadzhanov 2002), then that meteoroid was 0.18–0.35 m in diameter.

Large ejected grains can also be found around the cometary nuclei in pseudostable orbits. This constitutes a hazard for the spacecrafts visiting the neighbourhood of a comet (for physical risks of landing on a comet see Kuhrt, Knollenberg, & Keller 1997) and therefore the estimations of the size of the particles that can be orbiting around a cometary nucleus are of the greatest interest.

In this paper, I show in § 2 how the diameter of the largest ejected particle can be obtained. I use a classical outgassing theory with special attention to the rotation of the comet. I apply our expressions to comet 1P/Halley, a comet with a very low rotation period (§ 3), and to comet 46P/Wirtanen, a very fast spinning comet (§ 4). § 5 shows a comparison with results on the size of the ejected particles from several cometary nuclei obtained by radar measurements. Due to the interest in Comet 67P/Churyumov-Gerasimenko, the target of the Rossetta mission, I estimate the diameter of the largest ejected particle from the nucleus of the comet in § 6. The conclusions are presented in § 7.

2. SIZE OF THE LARGEST EJECTED GRAINS

Cometary gas of the nucleus drags dust particles from the surface of the comet. Some of those particles can be orbiting around the comet describing pseudostable orbits (Fulle 1997). Once the particles have left the nucleus surface, they suffer the gravitational forces of the nucleus and of the Sun, the solar radiation pressure force, the gas drag force and the inertial forces, which appear because a reference frame attached to the nucleus is considered. The equation of motion can be written as follows:

$$\frac{d^2\vec{r}_{\rm d}}{dt^2} = \left(\frac{3C_{\rm D}\dot{m}_{\rm g}v_{\rm g}}{16\pi C_{\rm p}Q_{\rm p}}\beta - GM_{\rm c}\right)\frac{\vec{r}_{\rm d}}{r_{\rm d}^3} + \frac{\mu M_{\rm S}G}{r_{\rm c}^3}\left(\frac{3\vec{r}_{\rm d}\cdot\vec{r}_{\rm c}}{r_{\rm c}^2}\vec{r}_{\rm c} - \vec{r}_{\rm d}\right) + \beta M_{\rm S}G\,\frac{\vec{r}_{\rm c}}{r_{\rm c}^3} + \Omega^2\vec{r}_{\rm d} - (\vec{\Omega}\cdot\vec{r}_{\rm d})\vec{\Omega} + 2\vec{v}_{\rm d}\times\vec{\Omega}\,. \quad (1)$$

Here, $\vec{r}_{\rm d}$ is the nucleus to grain vector, $C_{\rm D}$ is the drag coefficient, $\dot{m}_{\rm g}$ is the global gas mass loss rate (also appearing as Q in the literature), $v_{\rm g}$ is the gas velocity, $C_{\rm p} = 1.19\cdot10^{-3}$ kg m^{-2}, $Q_{\rm p}$ is the scattering efficiency of the grain, β is the ratio of radiation pressure to gravitational force of the Sun, G is the gravitational constant, $M_{\rm c}$ is the mass of the comet, $M_{\rm S}$ is the mass of the Sun, $\vec{r}_{\rm c}$ is the Sun to comet vector, $\mu = 1-\beta$, $\vec{\Omega}$ is the angular velocity of the rotation of the comet and $\vec{v}_{\rm d} = d\vec{r}_{\rm d}/dt$ where t is the time.

The drag coefficient depends on the shape of the particle and the flow conditions. Henderson (1976) presented, for spherical particles, accurate expressions for $C_{\rm D}$ varying over a wide range of flow conditions. In this paper, a spherical shape for the particle is assumed and a constant value of 2 for $C_{\rm D}$ is employed for all cases. The constant $C_{\rm p}$ is related to β by means of the expression $\beta = C_{\rm p}Q_{\rm p}(\rho_{\rm d}d)^{-1}$, where $\rho_{\rm d}$ is the density and d the diameter of the dust particle. Excluding very small grains, the radiation pressure force increases with decreasing size (Burns, Lamy, & Soter 1979). The value of $1.19\cdot10^{-3}$ kg m^{-2} is obtained from $C_{\rm p} = 3E_{\rm s}(8cGM_{\rm S})^{-1}$, where $E_{\rm s}$ is the mean total solar radiation and c the speed of light (see Finson & Probstein 1968). I also assume that the nucleus obliquity is 0. In the above expression we have used the same nomenclature as that employed by Fulle (1997) except for the last three terms, which are due to the rotation of the cometary nucleus, and are not present in the work of Fulle. However, they can be important as I show in this paper.

From equation (1), I obtain an expression for the particle with largest diameter that can be lifted from the surface (see, for example Molina, Moreno, & Jiménez-Fernández 2008):

$$d_{\rm max} = \frac{1}{\rho_{\rm d}g_{\rm eff}}\frac{3C_{\rm D}\dot{m}_{\rm g}v_{\rm g}}{16\pi R^2}\,, \quad (2)$$

where R is the radius of the cometary nucleus and $g_{\rm eff} = g - \Omega^2 R\cos^2\phi$ (g is the gravitational acceleration of the comet and ϕ is the latitude on the surface where the dust particle is located). In this paper, I show the importance of the rotation terms in the equation of motion, and, particularly, of the values obtained for $d_{\rm max}$, and I discuss the results obtained for several comets of different rotational periods. For comets rotating slowly, $\Omega^2 R\cos^2\phi \ll g$ and then $g_{\rm eff} \approx g$. As $g = G4\pi\rho_{\rm n}R/3$, it will be much larger than $\Omega^2 R\cos^2(\phi)$ for comets with $\tau^2 \gg 3\pi/\rho_{\rm n}G$,

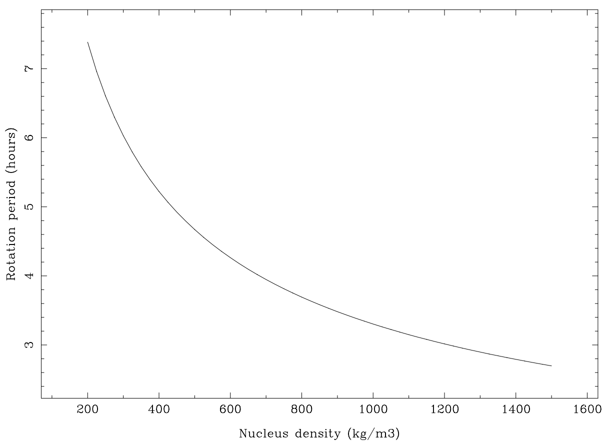

Fig. 1. Critical rotation period versus density of the cometary nucleus. Only the region above the curve is permitted.

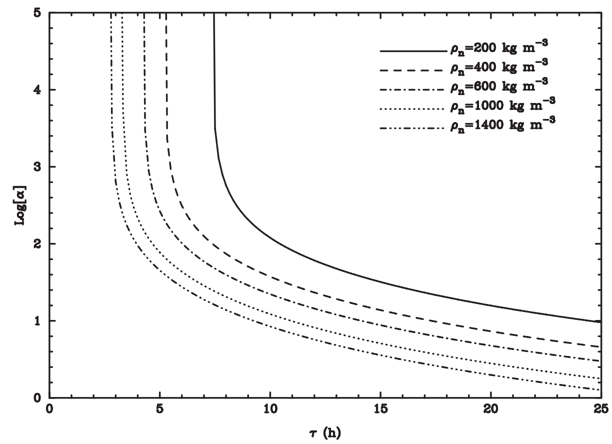

Fig. 2. Percentage relative increase in $d_{\rm max}$ due to cometary rotation for different values of the nucleus density.

where τ is the rotation period. For τ in seconds and $\rho_{\rm n}$ in kg m^{-3}, $d_{\rm max}$ will be independent of the nucleus rotation when $\tau^2 \gg 1.4 \cdot 10^{11} {\rho_{\rm n}}^{-1}$. Asuming the cometary nucleus as a strengthless body, $g_{\rm eff}$ must be positive and then only rotation periods larger than a critical period $\tau_{\rm crit} = \sqrt{3\pi/\rho_{\rm n} G}$ are possible, otherwise the comet would fly apart. Thus, $\tau_{\rm crit} = 3.76 \cdot 10^5 \frac{1}{\sqrt{\rho_{\rm n}}}$ with τ is in seconds and $\rho_{\rm n}$ is in kg m^{-3}. A curve of $\tau_{\rm crit}$ for different values of nucleus densities is shown in Figure 1.

In order to illustrate the relative importance of the cometary rotation in the obtained values of $d_{\rm max}$, I introduce the parameter α as the percentage relative increase in $d_{\rm max}$ due to the cometary rotation. Thus, $\alpha = 100 \cdot (d_{\rm max} - d^0_{\rm max}/d^0_{\rm max})$, where $d^0_{\rm max}$ is the value of $d_{\rm max}$ when the comet is not rotating. From equation (2), I obtain $\alpha = 100 \cdot (\frac{\rho_{\rm n} G \tau^2}{3\pi} - 1)^{-1}$. In Figure 2 I show α versus τ for different values of $\rho_{\rm n}$.

The quantity $\dot{m}_{\rm g} v_{\rm g}$ in the differential equation of motion is crucial, because the grains would not lift from the surface for low values of it (that is when the attractive cometary gravitational force is greater than the other forces acting on the grains). I assume that the emission of grains from the cometary nucleus is anisotropic and suggest a strong anisotropy towards the Sun. Following Fulle (1997), $\dot{m}_{\rm g} v_{\rm g} = 0$ if $\cos(Z) < 0$ (dark hemisphere) where Z the zenith angle, and if $\cos(Z) \geq 0$, $\dot{m}_{\rm g} v_{\rm g}$ varies as the third power of the cosine of the zenith angle. Then, I can write, for illuminated hemisphere, $\dot{m}_{\rm g} v_{\rm g} = P \frac{-\cos^3(Z)}{r^\gamma}$, where P is the normalization constant, and γ is a fit constant which depends on the comet. After integration over the illuminated hemisphere, I obtain $\dot{m}_{\rm g} v_{\rm g} = 8(\dot{m}_{\rm g} v_{\rm g})_0$, where $(\dot{m}_{\rm g} v_{\rm g})_0$ is the quantity $\dot{m}_{\rm g} v_{\rm g}$ for an isotropic emission. It should be noted that equation (2) is only valid at the sub-solar point of the nucleus if an anisotropic gas emissin in considered.

3. GRAIN EJECTION FROM A COMET WITH A HIGH ROTATION PERIOD. COMET 1P/HALLEY

The state of rotation of Halley's comet has been the subject of many works. An explanation of the two observed periods is to consider Halley's nucleus to be in a complex rotation. The long-axis rotates around the total angular momentum vector with a period of 3.69 days, and the component of spin around the long-axis has a period of 7.1 days (see Belton et al. 1991, and references therein). In any case, the observed rotation is slow enough to consider $g_{\rm eff} \approx g$.

Krankowsky et al. (1986) derived a water molecule density $n_{\rm H_2O} = 4.7 \cdot 10^7$ molecules cm^{-3} at 1000 km from the cometary nucleus from the neutral mass spectrometer experiment carried by the Giotto spacecraft and they estimated an uncertainty of 50%. The heliocentric distance of the comet was 0.89 AU. Since the derived water abundance was 80% by volume and since the volume mixing ratios (relative to H_2O) were 3.5% for CO_2, 10% (upper limit) for NH_3 and 7% (upper limit) for the CH_4, I obtain $Z_{\rm n} = 1.25 n_{\rm H_20} v_{\rm g}$, where $Z_{\rm n}$ is the gas mass flux expressed by the number of molecules cm^{-2} s^{-1}, and $v_{\rm g}$ is in cm s^{-1}. For an isotropic emission, I can obtain $\dot{m}_{\rm g}$ from $Z_{\rm n}$ multiplying by $4\pi R^2$. Krankowsky et al. obtained a gas expansion velocity (at distances closer than 10,000 km) of (900 ± 200) m s^{-1}. Putting this all together, and considering a strong anisotropic emission as reported by Fulle (1997), I conclude that $\dot{m}_{\rm g} v_{\rm g}$ ranges from $5.0 \cdot 10^7$ to $3.1 \cdot 10^8$ kg m s^{-2}.

Equation (2) can be rewritten as

$$d_{\rm max} = \frac{1}{\rho_{\rm d}} \frac{9 C_{\rm D} \dot{m}_{\rm g} v_{\rm g}}{64\pi^2 R^3} \frac{1}{G\rho_{\rm n} - \frac{3\pi}{\tau^2}\cos^2\phi} \,. \qquad (3)$$

Using $\dot{m}_{\rm g} v_{\rm g} = 5.0 \cdot 10^7 - 3.1 \cdot 10^8$ kg m s^{-2} in equation (3), and considering that $\frac{3\pi}{\tau^2}\cos^2\phi \ll G\rho_{\rm n}$ (in this case of comet Halley), I obtain values of $d_{\rm max}$ between 0.17 and 1.06 m if dust and nucleus densities are equal to 1000 kg m^{-3}.

4. GRAIN EJECTION FROM A COMET WITH LOW ROTATION PERIOD. COMET 46P/WIRTANEN

As mentioned in the introduction, a purpose of this paper is to show the importance of nucleus rotation for the values obtained for $d_{\rm max}$. For that reason, I apply the previous equations to the case of a comet with a short rotation period, such as comet 46P/Wirtanen. This comet was discovered by C. A. Wirtanen in January 1948 by studying its proper motion on plates obtained at the Lick Observatory with the 20 inch f/7.4 Carnegie astrograph. Although this comet is not now a target of any space mission (it was initially chosen as a target of Rosetta) it is among the best observed short period comets and has been studied from the ultraviolet to radio wavelengths from ground and from space (see Schulz & Schwehm 1999, and references therein). Furthermore, I have choosen this comet because its rotation period of 6 hours makes it one the fastest rotators among cometary nuclei having a well-determined rotational period (Lamy et al. 1998).

Comet 46P/Wirtanen is a small comet of 0.60 ± 0.02 km radius as derived by Lamy et al. (1998) from Hubble Space Telescope measurements. This radius is smaller than that reported by Crifo & Rodionov (1997) which is 1.4 km. The former authors considered their obtained values for the nucleus radius more realistic owing to the much higher contrast between nucleus and coma in the HST observation than that displayed in the ground-based observations by the latter authors.

Assuming a nucleus mass density of 1000 kg m^{-3} for Comet 46/P Wirtanen and considering a value for the rotation period of 6 hours (Lamy et al. 1998), I obtain a factor α of 43. The largest particle ejected from the nucleus due to the fast rotation of comet Wirtanen has a diameter more than 40 percent larger than the non-rotating case. This result is in agreement with Groussin & Lamy (2003), who pointed out that the maximum size must be enlarged by a factor of about 1.4 if the rotation of the comet is taken into account. In order to obtain absolute values for $d_{\rm max}$ I must know $\dot{m}_{\rm g} v_{\rm g}$. The evolution of Comet 46P/Wirtanen was investigated as a function of heliocentric distance at $r_{\rm h} \leq 4.6$ AU (see, e.g., Schulz & Schwehm 1999). The composition of the emitted gas was mainly water (more than 99 percent) near perihelion. Therefore, the number of water molecules per second is essentially the gas production rate. Enzian (1999) gives an upper limit for the CO outgassing rate based on model results of less than 10^{26} molecules s^{-1}. Even assuming an abundance of CO and CO_2 molecules of 1%, the value of $d_{\rm max}$ would only be affected by about 2%. A value of about $(2.0 \pm 0.4) \cdot 10^{28}$ water molecules per second near perihelion seems to be a reasonable value for the gas production rate of 46P/Wirtanen (Fink & Combi 2004) although values as large as $4 \cdot 10^{28}$ (Jorda & Rickman 1995) and as small as $1 \cdot 10^{28}$ (A'Hearn et al. 1995) also appear in the literature.

In order to determine the value for $v_{\rm g}$, I apply the expression for the thermal velocity $v_{\rm th}$ and $v_{\rm g} = (1/2) v_{\rm th}$ (Wallis 1982) and then, $v_{\rm g} = \sqrt{\frac{2RT}{\pi M}}$, where R the ideal gas constant, T the temperature and M the water molecular weight. Crifo & Rodionov (1997) report that total gas production rates of 4×10^{28} molecules per second or greater unquestionably produce hydrodynamical flows. A detailed discussion of the outflow speed of the gas is made by Ma, Williams, & Chen (2002), who argued that a fluid model is adequate for describing many of the phenomena associated with the cometary gas tail, but that an approach based on the kinetic theory of gases must be used for individual grain interactions. I use here the above expression for the gas velocity, which is the same thermal expansion velocity used by Nolan et al. (2006) (In the next section I compare our results with those obtained by Nolan et al. from radar measurements). Assuming a value for T of about 200 ± 100 K, I obtain $v_{\rm g} = 240$ m s^{-1} with an upper limit of 300 m s^{-1} and a lower limit of 170 m s^{-1}. Taking into account the considerations shown above I estimate a value for $\dot{m}_{\rm g} v_{\rm g}$ between $0.8 \cdot 10^5$ and $21.5 \cdot 10^5$ kg m s^{-2} for an isotropic gas emission. Then, after considering equation (3), using $R = 600$ m and $\rho_{\rm d} = 10^3$ kg m^{-3} I obtain values for $d_{\rm max}$ between 0.22 and 6.1 m.

Fulle (1997) also considered for 46P/Wirtanen an ejection varying as the third power of the cosine of the zenith angle, as indicated in the Halley case. If such a strongly anisotropic gas emission is assumed the upper limit for the diameter of the boulder could be several tens of meters. A representative value for $d_{\rm max}$ should be $d_{\rm max} = 0.4$ m, which is obtained for a total gas production rate of $2 \cdot 10^{28}$ water molecules per second when the comet was at ~ 1.1 AU of heliocentric distance, $v_{\rm g} = 240$ m s^{-1}, $R = 600$ m, $\rho_{\rm d} = 10^3$ kg m^{-3} and a 6 hour period, assuming an isotropic gas emission. If a strongly anisotropic emission is assumed (as the third power of the cosine of the zenith angle) our value would be $d_{\rm max} = 3.2$ m. An intermediate value could be obtained if the ejection varies with the cosine of the zenith angle (as assumed by Crifo & Rodionov 1997).

5. COMPARISON WITH OTHER COMETS OBSERVED BY RADAR

Radar observations have indicated the presence of large grains in several comets (see Harmon et al. 2004, and references therein). A firm detection of large grains from radar measurements was first published in Nolan et al. (2006), when they analyzed radar measurements of Comet C/2001 A2 (LINEAR). Besides this comet, Nolan et al. showed results for all comets for which a grain coma was detectable by radar. These comets are: C/2001 A2 (LINEAR), Halley, IRAS-Araki-Alcock, Hyakutake, and C/2002 O6, although the last comet was not modeled because the radar observations presented a low signal-noise ratio. They computed the Doppler spectra from the radial components of the terminal velocities of the entire ensemble of grains within the radar beam. They used a gas-drag model, first proposed by Whipple (1951) which postulates the dragging of particles by outflowing gases released by sublimation from the surface of the cometary nucleus. This model leads to an expression for the terminal ejection velocity of a spherical particle: $V_{\rm t} = V_0(1 - a/a_{\rm m})^{1/2}$, where V_0 is the terminal velocity in the absence of gravity, a is the radius of the particle, and $a_{\rm m}$ is the radius of the largest grain that can be lifted (see Harmon et al. 1989, and references therein). Also, $V_{\rm t}$ is expressed by $V_{\rm t} = C_{\rm v} a^{-1/2}(1 - a/a_{\rm m})^{1/2}$ where $C_{\rm v} = V_0 a^{1/2}$) is a velocity scale factor. Fitting the model to the radar Doppler measurements they obtained values for the factor $C_{\rm v}$. From this model $C_{\rm v} = (3C_{\rm D} v_{\rm g} Z R / 4\rho_{\rm d})^{1/2}$, where Z is the mass gas flux at the surface; the model also leads to the equation (4) by Nolan et al. (2006),

$$a_{\rm m} = \frac{9 C_{\rm D} v_{\rm g} Z}{32\pi G R \rho_{\rm n} \rho_{\rm d}} = \frac{3 {C_{\rm v}}^2}{8\pi G R^2 \rho_{\rm n}} \,. \quad (4)$$

Nolan et al. get a good fit to the spectra with $C_{\rm v} = 36$ cm$^{1/2}$ m s^{-1} for the comet C/2001 A2 LINEAR. They assumed $\rho_{\rm n} = 1000$ kg m^{-3}, $\rho_{\rm d} =$ 500 kg m^{-3}, $R = 1$ km, and a temperature at the surface $T = 250$ K. Thus, they obtained a value for Z of $7 \cdot 10^{-4}$ g cm^2 s^{-1} and a value for $a_{\rm m} = 10$ m. However, if $C_{\rm v} = 36$ cm$^{1/2}$ m s^{-1} is inserted in their equation (4), other values, in fact, are obtained: $Z = 1.6 \cdot 10^{-3}$ g cm^2 s^{-1}, $a_{\rm m} = 23.2$ m. For the moment I do not know the reason of such discrepancy.

Woodney, Schleicher, & Greer (2001) presented results from photometry and narrow band imaging of Comet C/2001 A2 (LINEAR) obtained at the Lowell Observatory 1.1 m Hall telescope. They studied the outward motion of the CN arcs and concluded that a 3 or 6 hour rotational period was possible, but they had too few data to obtain a unique solution (D. G. Schleicher, personal communication, May 19, 2009). If a 6 hour period is considered, then the inclusion of the rotational effects should enlarge the estimated maximum radius of the lifted particles by a factor of 1.43 (as in the case of comet 46P/Wirtanen). Therefore, the $a_{\rm m}$ estimated by Nolan et al. (2006) should be more than 14 m. I do not consider here the possible period of 3 h because it would be less than the critical period for a nucleus density of 1000 kg m^{-3} as can be seen in Figure 1, and for that value of the period, a high nucleus density ($\rho_{\rm n} > 1200$ kg m^{-3}) should be assumed. Besides, the fastest cometary rotators observed have rotational periods larger than 5 hours (e.g., Meech 1996).

In order to compare the $d_{\rm max}$ values obtained from radar measurements with other techniques I consider the molecular abundances obtained by Magee-Sauer et al. (2008) on 10 July 2001 using the NIRSPEC instrument on the Keck-2 telescope at Mauna Kea. As the composition is not only water (although water molecules account for nearly 90% of the total molecular abundance), I must proceed in a similar way as for comet Halley. Thus, I take the average values for the July observations (Table 4 in Magee-Sauer et al. 2008): [402(H_2O), 6.3(C_2H_6), 16.6(CO), 13(CH_3OH), 5.5(CH_4), 1.6(C_2H_2), 2(HCN), 0.2(H_2CO)] $\cdot 10^{26}$ molecules per second at a heliocentric distance of $1.160 - 1.173$ AU. The result is $\dot{m}_{\rm g} = 1.4 \cdot 10^3$ kg s^{-1}. This value is in agreement with that reported by Nolan et al. (2006) assuming isotropic grain ejection: $1 - 3 \cdot 10^3$ kg s^{-1}. It is an excellent result because both values were obtained using completely different techniques. Assuming a gas temperature of 250 K, I obtain a value of 270 m s^{-1} for $v_{\rm g}$. Then, inserting $\dot{m}_{\rm g} = 1.4 \cdot 10^3$ kg s^{-1} and $v_{\rm g} = 270$ m s^{-1} in equation (2) and omitting the rotation term, I obtain $d_{\rm max} = 0.32$ m for isotropic ejection. If I consider an anisotropic ejection with third power of the cosine of the solar zenith angle and the fast rotation of the Comet C/2001 A2 (six hours period) I obtain a value $d_{\max} = 3.6$ m, which is a large value but much smaller than that the one reported by Nolan et al. (2006) using $C_{\rm v} = 36$ cm$^{1/2}$ m s^{-1}.

The other two comets mentioned above have a very different rotation period. IRAS-Araki-Alcock (comet IAA) is a very slow rotator. Feldman, A'Hearn, & Millis (1984) observed the ultraviolet emissions from comet IAA and showed that the rotation period must be ≥ 27 h. Harmon et al. (1989) derived a nucleus rotation period of 2–3 d from radar measurements. Then, $g_{\rm eff} = g$ and the value of $d_{\max}$ is unaffected by the rotation term of equation (3). Harmon et al. (1989) estimated values for the maximum radius $a_{\rm m}$ of the ejected particles of 3 cm from radar polarization measurements. They also applied a gas-drag model assuming an isotropic ejection and obtained $a_{\rm m} = 6 \times 10^{-4}$ m, which is inconsistent with the result obtained from the radar measurements. Then, they argued like Hanner et al. (1985) that is more likely that only 1% of the total surface area is active, and therefore a factor of 50 must be applied to the value $a_{\rm m} = 6 \times 10^{-4}$ m obtained for an isotropic emission on the illuminated hemisphere. Thus a value of $a_{\rm m} = 0.03$ m is found, which is consistent with the result derived from radar measurements.

On the other hand, I have used equation (2) of this paper and the assumption of an emission as $\cos^3(Z)$ and I obtained $d_{\max} = 5 \times 10^{-3}$ m. I have used $\rho_{\rm n} = \rho_{\rm d} = 10^3$ kg m^{-3}, $v_{\rm g} = 280$ m s^{-1}, $R = 5$ km, and the gas mass-loss rate of $6 \cdot 10^5$ g s^{-1} measured by Feldman et al. (1984) at 1.024 AU. All these values are those also used by Harmon et al. (1989). If I insert the value of $C_{\rm v} = 8$ cm$^{1/2}$ m s^{-1} as shown by Nolan et al. (2006) in their equation (4) I obtain a value of $a_{\rm m} = 0.06$ m, which is inconsistent with the value $a_{\rm m} = 6 \times 10^{-4}$ m reported by Harmon et al. (1989).

Comet Hyakutake (C/1996 B2) was also observed by radar on March 1996 by Harmon et al. (1997) at a heliocentric distance of $\simeq 1$ AU. They observed centimeter-sized grains in the coma of the comet. As the coma echo was asymmetric, they proposed that the asymmetry implied an anisotropy in the direction of grain ejection, and assumed that the grains were ejected in a 90°-wide cone. There are several estimations of the nucleus size of this comet based on infrared observations and on radio continuum measurements (see cited references in Harmon et al. 1997) and a value of about 2.4 km of radius is representative. Taking for the radius of the comet $R = 2.4$ km, $\rho_{\rm n} = 1000$ kg m^{-3}, and the same values as Harmon et al. (1997), $\rho_{\rm d} = 300$ kg m^{-3}, $v_{\rm g} = 290$ m s^{-1}, and $\dot{m}_{\rm g} = 5 \cdot 10^3$ kg s^{-1}, I obtain from equation (2) $d_{\max} = 0.15$ m if isotropic ejection is assumed. At first, two possible periods were attributed to Hyakutake (C/1996 B2). Schleicher et al. (1996) obtained a 6 h 14 min period from narrow-band photoelectric photometry and Larson et al. (1996) derived a rotation period of 12.5 h from observations made with the Hubble Space Telescope. However, Schleicher et al. (1998) presented a definitive determination of Comet Hyakutake's rotational period of 6.23 ± 0.03 h combining photometric data with CCD images. Then, and after adopting this last value, the rotation factor α is 39. I have also assumed (as Harmon et al. 1997) that the grains are ejected asymmetrically, although with a stronger asymmetry. I have maintained our assumption of an emission as $\cos^3(Z)$, for comparison, and then I obtain $d_{\max} = 1.7$ m. On the other hand, if the value $C_{\rm v} = 40$ given by Nolan el al. (2006) is inserted in their equation (4), then $a_{\rm m}$ near 5 m is found, which is not consistent with the results by Harmon et al. (1997).

6. COMET 67P/ CHURYUMOV-GERASIMENKO

This comet is the current target for the Rosetta mission. The Rosetta spacecraft will rendezvous and land upon the surface of comet Churyumov-Gerasimenko in late 2014. That is why it is important to consider the $d_{\max}$ obtained with the present model. Although first estimations favored a radius of about 3 km, more precise studies indicate a radius of about 2 km. Lamy et al. (2007) determined the size and shape of the nucleus of this comet from several visual light curves. They concluded that the comet is an irregular body with an effective radius of 1.72 km, and they found that it is rotating around a principal axis with a period of 12.4–12.7 hours. They estimated a nuclear density of 370 kg m^{-3} and densities between 100 and 500 kg m^{-3} were reported by Davidsson & Gutiérrez (2005). Tubiana et al. (2008) performed broad-band imaging of 67P/Churyumov-Gerasimenko in the visual range with a mosaic of two 2K×4K MIT CCDs at the 8.2 m Very Large Telescope UT1 and determined a rotation period of 12.7407 ± 0.0011 h and an effective radius of 2.38 ± 0.04 km. Lamy et al. (2008) and Kelley et al. (2009) using the Multiband Imaging Photometry (MIPS) 24 μm channel of the Spitzer Space Telescope obtained a mean effective radius of $1.93-2.03$ km and 2.04 ± 0.11 km, respectively. Here, I adopt a value of 2 km for the radius of the nucleus of

TABLE 1

VALUES OF $d_{\rm max}$ FROM RADAR MEASUREMENTS COMPARED WITH THOSE FROM THIS WORK[a]

Comet	Radar measurements		This work		
	$d_{\rm max}$ (m)	$r_{\rm c}$ (AU)	$d_{\rm max}$ (m)	$r_{\rm c}$ (AU)	α
Halley	≥ 0.04[b]	1.59–1.47	0.17–1.06	0.89	$\simeq 0$
Hyakutake	10[c]	$\simeq 1$	1.7	$\simeq 1$	39
C/2001 A2	20[d]	$\simeq 1.14$	3.6	$\simeq 1.16$	43
IAA	0.06[e]	$\simeq 1$	0.005	$\simeq 1$	$\simeq 0$
Wirtanen			3.2	1.1	43
67P/C-G			0.3	1.34	29

[a]The last column shows the percentage relative increase in $d_{\rm max}$ due to cometary rotation (see text). [b]Campbell, Harmon, & Shapiro (1989). [c]Harmon et al. (1997). [d]Nolan et al. (2006). [e]Harmon et al. (1989).

the comet 67P/Churyumov-Gerasimenko. Lamy et al. (2007) carried out production rate measurements during both the 1982 and the 1996 apparition, and obtained approximate least-squares fitted analytical curves. I consider a gas production rate of $0.7 \cdot 10^{28}$ molecules per second, which is the maximum value of the above mentioned fitted curve near perihelion and, according to equation (4) of that paper it corresponds to a heliocentric distance of 1.34 AU. Agarwal, Müller, & Grün (2007) obtained terminal speeds of dust particles as functions of size at perihelion ($\simeq 1.29$ AU). Their values go from several hundreds of meters per second for submicrometer particles to a few meters per second for centimeter particles. Here I assume $v_{\rm g} = 270$ m s^{-1}, which is the value corresponding to a temperature of 250 K using the kinetic theory and of the same order as that used by Nolan et al. (2006, and references therein) for comparison. I assume the mean value of the interval mentioned by Davidsson & Gutiérrez (2005) for $\rho_{\rm n} = 300$ kg m^{-3}, $\rho_{\rm d} = \rho_{\rm n}$ in the absence of published information for $\rho_{\rm d}$, and $\tau = 12.7407$ h. Then, I obtain a value of $d_{\rm max} = 0.3$ m, if an ejection as the third power of the cosine of the zenith angle is considered. The centrifugal force due to the 12.7407 h rotation period increases the value of $d_{\rm max}$ as $\alpha = 28.8$. I show in Table 1 our $d_{\rm max}$ results together with those obtained from radar measurements (Nolan et al. 2006, and references therein) for comparison. I would like to point out that the values obtained for $d_{\rm max}$ are referred to a particular comet-to-Sun distance, because $\dot{m}_{\rm g} v_{\rm g}$ depends on $\vec{r}_{\rm c}$.

7. CONCLUSIONS

The standard model of planetary outgassing constitutes an useful way to estimate the maximum size $d_{\rm max}$ of the dust lifted from the surface of a comet. I have compared our results with those obtained from radar measurements for four comets: Halley, Hyakutake, C/2001 A2 (LINEAR) and IRAS-Araki-Alcock. In general, our results are smaller than those reported by Nolan et al. (2006, and references therein). Additionally, I have estimated values of $d_{\rm max}$ for comet 46P/Wirtanen and for comet 67P/Churyumov-Gerasimenko. The uncertainties in the quantities of equation (3) can considerably affect the estimated values of $d_{\rm max}$. Mainly, $\dot{m}_{\rm g} v_{\rm g}$ and R must be well known. The radius of the nucleus R appears as the third power in equation (3) and, on the other hand, the quantity $\dot{m}_{\rm g} v_{\rm g}$ is usually not well known. The discrepancy between the values of $v_{\rm g}$ obtained by using kinetic theory and those from the hydrodynamic models will be resolved when the GIADA (Grain Impact Analyser and Dust Accumulator) experiment measures the velocity of the grains from comet Churyumov-Gerasimenko. Independently of these uncertainties, $d_{\rm max}$ will be increased due to rotation of the nucleus. Thus, $d_{\rm max}$ increases by more than 40 percent due to rotation for fast comets with periods around 6 h. Therefore, the inertial forces, which introduce rotation terms in the equation of the cometary dust ejection, cannot be ignored, unless the comet is rotating with long a period (≥ 15 hours, $\alpha \leq 5$ for $\rho_{\rm n} = 10^3$ kg m^{-3}).

I have considered a strong anisotropy toward the Sun (as the third power of the cosine of the zenith angle) as proposed by Fulle (1997). This is equivalent to assuming an emission by an active area of about 12% of the total area, instead of assuming isotropic ejection. A more moderate assumption considering an anisotropy toward the Sun as the cosine of the zenith angle would lead to values of $d_{\max}$ just half of those estimated here. The dependence on the heliocentric distance, eccentricity of the orbit of the comet, true anomaly, obliquity and longitude of the place where the grain is lifted, will be studied in the future.

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A. Molina: Departamento de Física Aplicada, Universidad de Granada, Facultad de Ciencias, Avenida Severo Ochoa s/n, 18071, Granada, Spain and Instituto de Astrofísica de Andalucía, CSIC, Camino Bajo de Huétor 50, 18008, Granada, Spain (amolina@iaa.es).