Jones 2001; Franco et al. 2002; Kosiński & Hanasz 2006, 2007; Lee & Hong 2007; Mouschovias et al. 2009).

The structure of the final static state resulting from the Parker instability was derived by Mouschovias (1974, hereafter M74) in the Cartesian model, namely, for a non-rotating layer under a vertical gravity, with a horizontal magnetic field in the initial equilibrium state. In the unperturbed state, the gas is in hydrostatic equilibrium in a stratified layer, so that all the quantities depend only on the distance from the midplane y, and the magnetic field is plane-parallel and runs along the x-direction. In this *two-dimensional Cartesian* problem, one assumes that all the quantities are independent of the third dimension, z, at any time. In the final state, the magnetic field rises in certain regions and sinks in others forming bulges and valleys. Matter loads onto the field lines by draining down from the top region of the bulge and sinking into the valleys.

So far, the final states of the Parker instability have been discussed in Cartesian models. In various astrophysical systems, however, the gas tends to form structures with other symmetries, such as filaments or elongated structures (e.g., Genzel & Stutzki 1989; Alfaro, Cabrera-Caño, & Delgado 1992; Ryu, Kang, & Biermann 1998; Conselice, Gallagher, & Wyse 2001; Salomé et al. 2006; Jackson et al. 2010). Suppose now that we have a very elongated self-gravitating filamentary structure with a magnetic field traveling along the major axis of this structure, such that it has the effective geometry of a infinite cylinder. If the filament is supported against the radial self-gravity of the system by the pressure of gas and the longitudinal magnetic field, long wavelength deformations of the magnetic field lines will allow matter to slide radially along field lines under the action of the gravitational field towards the symmetry axis, whereas the magnetic field will tends to expand in radius, through a process of magnetic buckling (see Figure 1). If the disturbances are axisymmetric, we expect the alternate formation of magnetic bottlenecks in the condensations of gas separated by portions of inflated magnetic lines or arcades, resembling a sausage mode. The final state is expected to be different from than of a vertically stratified disk such as those models currently used to study the Parker instability in galactic disks. In fact, the response of the system and the final equilibrium state depend on the adopted model: Cartesian or axisymmetric.

In this paper, we study the nonlinear final equilibria of the Parker instability of a non-rotating,

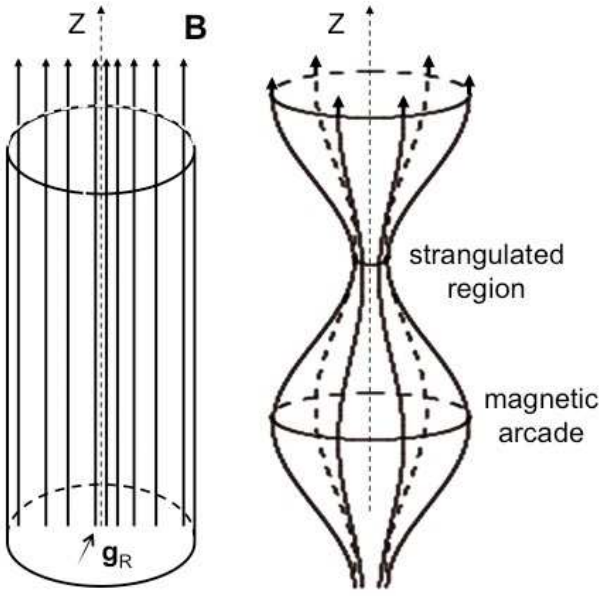

Fig. 1. Sketch of the magnetic field configuration in the initial (left panel) and final (right panel) equilibrium states in axial symmetry.

cylindrical initial state. In the initial state, the magnetic field is assumed to be parallel to the axis of the cylinder, which is taken to coincide with the z-axis. The cylinder is supported against the action of a radial gravitation field g_R by the thermal and magnetic pressures (see Figure 1). We will assume that the gas is isothermal and evolves under flux-freezing conditions. For axisymmetric perturbations, all the quantities depend only on z and R (not on the azimuthal angle ϕ) and hence the problem is also two-dimensional. The evolution of the Parker instability in the (z, R)-plane is expected to be qualitatively analogous, in the main features, to the evolution found in the (x, y)-plane for the case of a plane-parallel initial state by M74. Throughout the paper, we will use the coordinate z to denote the distance on the axis of symmetry in the axisymmetric model; it should not be confused with the third component of the Cartesian model.

The aim of this work is to compare in detail how the final distributions of density and magnetic field vary from Cartesian to axisymmetric geometry[3]. Accurate predictions of the nonlinear final states, an interesting problem in itself, can be very useful for the purpose of testing MHD codes. We will follow Mouschovias' procedure (1974, 1976) to construct the final equilibrium states. Nakano (1979)

[3]Note that the change in the geometry of the system is, by no means, equivalent to a coordinate transformation.