the Parker instability develops in time and parts of the magnetic field lines bulge outward until the tension of the field lines increases sufficiently for an equilibrium state to be established. In the final states arising from the Parker instability, the magnetic field lines are curved and the magnetohydrostatic force equation is

$$-\vec{\nabla}P - \rho\vec{\nabla}\psi + \frac{1}{c}\vec{J}\times\vec{B} = 0, \tag{5}$$

where $\vec{J}$ is the current density. The coordinate symmetries of the unperturbed states (see § 2) reduce the problem to the (x, y)-plane in the Cartesian model and to the (z, R)-plane in the axisymmetric model. Under these symmetries, flux-freezing allows to transform the magnetostatic equations of equilibria into a second-order elliptic partial differential equation (Dungey 1953), conventionally called the Grad-Shafranov equilibrium equation. The beauty of this approach is that the final equilibrium states can be found with no need to solve a time-dependent problem (M74; Mouschovias 1976). In the following we give the Grad-Shafranov equilibrium equations and the boundary conditions. Details on its derivation can be found in M74, Mouschovias (1976) and Nakano (1979).

3.1. *The Cartesian model*

In the Cartesian layer with one ignorable coordinate, so that all quantities depend only on (x, y), the magnetic vector potential can be written as $\vec{A} = A(x, y)\hat{e}_z$ and then $B_x = \partial A/\partial y$ and $B_y = -\partial A/\partial x$. One can show that the scalar function $q(x, y)$, defined by

$$q = P\exp\left(\frac{\psi}{c_s^2}\right), \tag{6}$$

is constant on a field line *at magnetohydrostatic equilibrium*, and is given by:

$$q(A) = \frac{c_s^2}{2}\frac{d\tilde{m}}{dA}\left[\int_0^{\lambda_x/2} dx \frac{\partial y(x, A)}{\partial A}\exp\left(-\frac{\psi(x, A)}{c_s^2}\right)\right]^{-1}, \tag{7}$$

where λ_x is the perturbation wavelength along the initial magnetic field and $d\tilde{m}$ is the mass per unit length (along the ignorable coordinate) in a flux tube between field lines characterized by A and $A + dA$ and between $x = 0$ and $x = \lambda_x$ (i.e. the mass-to-flux ratio). Under flux-freezing conditions, the mass-to-flux ratio is a constant of motion and, therefore, can be determined from the initial equilibrium configuration:

$$\frac{d\tilde{m}}{dA} = \lambda_x\frac{\rho_0(A)}{B_0(A)}. \tag{8}$$

Finally, the magnetic equilibrium equation (5) may be written in terms of A as

$$\nabla^2 A = -4\pi\frac{dq}{dA}\exp\left(-\frac{\psi}{c_s^2}\right), \tag{9}$$

(Dungey 1953; M74). Once the boundary conditions are specified, equations (7) and (9) can be solved numerically by using an iterative scheme (M74).

The boundary conditions we take are the same as those in M74. The system is assumed to be periodic in x and symmetric about the x-axis so that equation (9) is solved in the rectangle $0 < x < X_{\max}$ with $X_{\max} = \lambda_x/2$ and $0 < y < Y_{\max}$, with the boundary conditions

$$\left.\frac{\partial A}{\partial x}\right|_{x=0, X_{\max}} = 0. \tag{10}$$

The upper boundary must be far enough from the midplane to not affect the evolution, since a small vertical size of the computational domain suppresses the instability (e.g., M74; Mouschovias 1996). At the upper boundary we impose the condition that the field lines are not deformed, $A(x, Y_{\max}) = A_0(Y_{\max})$. Due to the imposed reflection symmetry about the midplane, the field line originally coinciding with the x-axis remains undeformed.

In the following, we provide the equations for the particular case of a constant external field. Although a uniform acceleration is only realistic at high altitudes from the midplane, we will concentrate on this case for simplicity and to facilitate comparison with some previous work that uses a constant gravitational field (e.g., M74; Mouschovias et al. 2009). A realistic model of the gravitational potential is important to derive the growth rate of the instability and the most unstable wavelength, which determines the separation of the magnetic valleys (Kim & Hong 1998; § 4.2). For now, we are mainly interested in the main conceptual features that arise in the final equilibria states when the geometry of the problem is changed, even if the adopted spatial dependence of the gravitational field lacks a strong astrophysical motivation.

As said in § 2, if the external gravitational field is constant, the gravitational potential is $\psi = g|y|$, with g a positive and constant acceleration. The equations are put in dimensionless form by choosing the following natural units: c_s^2/g, c_s and $\rho_0(0)$, for length, velocity and density, respectively. The magnetic vector potential will be measured in units of $-2HB_0(0)$. Note that the dimensionless vertical scale is $H = (1+\alpha)$ in these units. The dimensionless equation (9) has the form

$$\nabla^2 A = \hat{Q}_{xy}(x, y; \alpha), \tag{11}$$