of the differential equation (21). In a model with uniform radial gravitational field, the magnetic flux and the q-function in the unperturbed state are, respectively:

$$\Phi_0(R) = \left(1 + \frac{R}{2(1+\alpha)}\right) \exp\left(-\frac{R}{2(1+\alpha)}\right), \quad (26)$$

and

$$q_0(R) = \exp\left(\frac{\alpha R}{1+\alpha}\right). \quad (27)$$

At small enough R, say $R \ll 2(1+\alpha)$, they can be approximated by $\Phi_0(R) \approx \exp(-R/2[1+\alpha])$ and $q_0(\Phi_0) = \Phi_0^{-2\alpha}$. Interestingly, they have exactly the same forms as $A_0(y)$ and $q_0(A)$ in the Cartesian model, respectively (see equation 15 and the very end of § 3.1). At large radii, $R \gg 2(1+\alpha)$, or equivalently, $\ln q \gg 2\alpha$, the relationship between Φ_0 and q_0 is $\Phi_0 \simeq (2\alpha)^{-1} q_0^{-1/(2\alpha)} \ln q_0$. Therefore, at these large q values, q increases with Φ^{-1} slightly faster than the rate at which q increases with A^{-1} in the Cartesian model ($q = A^{-2\alpha}$). In § 4, we will calculate the dependence of the function q on Φ in the final axially-symmetric equilibrium state. The shape of $q(\Phi)$ as compared to $q(A)$ in the Cartesian model will be also discussed.

The source term of the differential equation in the initial cylindrical configuration with constant g_R is:

$$Q_{zR}^{(i)} = \frac{1}{8(1+\alpha)^3} R \exp\left(-\frac{R}{2(1+\alpha)}\right). \quad (28)$$

By comparing equations (16) and (28), we see that $Q_{zR}^{(i)}$ has a similar dependence on R as $\hat{Q}_{xy}^{(i)}$ on y, except by the factor $R/(2+2\alpha)$. As a consequence, $Q_{zR}^{(i)}$ has a maximum at $2(1+\alpha) = 2H$, whereas $\hat{Q}_{xy}^{(i)}$ is a monotonically decreasing function of y. In order to facilitate comparison between both cases, it is convenient to use Q_{xy} defined as $Q_{xy} \equiv y\hat{Q}_{xy}/(2+2\alpha)$ because both Q_{xy} and Q_{zR} are identical at the initial state. However, they are expected to depart from each other in the nonlinear equilibrium state because of the changes introduced by the different geometries. The form of Q_{xy} and Q_{zR} in the final equilibrium states will be discussed in the next section.

4. FINAL EQUILIBRIUM STATES

The final equilibrium state can be reached from the initial configuration through continuous deformations of field lines using an iterative method as described in M74. To do so, we add a perturbation to the magnetic variables in the initial equilibrium state,

$$\delta A(x,y) = -\mu A_0(y) \sin\left(\frac{\pi y}{Y_{\max}}\right) \cos\left(\frac{\pi x}{X_{\max}}\right), \quad (29)$$

for the Cartesian model and

$$\delta\Phi(z,R) = -\mu \Phi_0(R) \sin\left(\frac{\pi R}{R_{\max}}\right) \cos\left(\frac{\pi z}{Z_{\max}}\right), \quad (30)$$

for the axisymmetrical model. Here μ is the amplitude of the perturbation which we chose between 0.02 and 0.06. Note that the perturbations are symmetric about $y = 0$ in the Cartesian model[4].

Our main interest is to compare the final equilibrium states in Cartesian and axisymmetrical models using the same external gravitational potential along y and R, respectively. In this way, the comparison can be done on a common ground. In the next subsection, we will do so for models with constant gravitational acceleration. A more realistic gravitational potential will be studied in § 4.2 in order to examine how it might affect the final nonlinear states.

4.1. *Constant gravitational acceleration*

Parker (1966) showed that a plane-parallel atmosphere with uniform gravity and a horizontal magnetic field is unstable to the undular mode if the horizontal wavelength of the perturbation is larger than a critical value, namely, $\lambda_x > \lambda_{\rm crit} \equiv 4\pi(1+\alpha)(2\alpha+1)^{-1/2}$. For the reference value $\alpha = 1$, the dimensionless critical wavelength is 14.5 or, equivalently, $7.26H$. Magnetic tension can stabilize disturbances with shorter wavelengths. The horizontal wavelength of the mode with the maximum growth rate depends on the vertical wavelength. For a typical value of $Y_{\max} = 24$, the maximum growth rate occurs at $X_{\max} = 12$ (i.e. $\lambda_x = 24$).

The stability of magnetized cylinders has been considered in the past (e.g., Stodolkiewicz 1963; Nagasawa 1987) but, to our knowledge, none derived the dispersion relation under the assumption of a constant gravitational field. Nevertheless, the linearized equations of motion under axial geometry and axisymmetric perturbations are identical to equations (III-3)–(III-6) quoted in Parker (1966), once the subscripts y and z in Parker's notation are replaced by z and R, except for the curvature term v_R/R that appears as an extra term in the continuity and energy equations. Since the curvature term

[4]The Cartesian model allows also solutions where field lines cross the midplane. They are called odd-parity solutions or midplane antisymmetric modes.