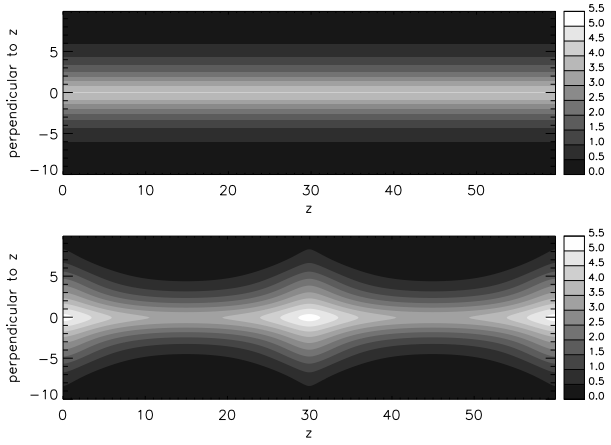

Fig. 7. Column density map for the initial (top panel) and final axisymmetric state (bottom panel) along a line of sight perpendicular to the axis of symmetry (z-axis). We used $\alpha = 1$, $\lambda_z/2 = 15$ and uniform radial gravity. The units of surface density and length are $\rho_0(0)c_s^2/g$ and c_s^2/g, respectively. Note that the scale is linear.

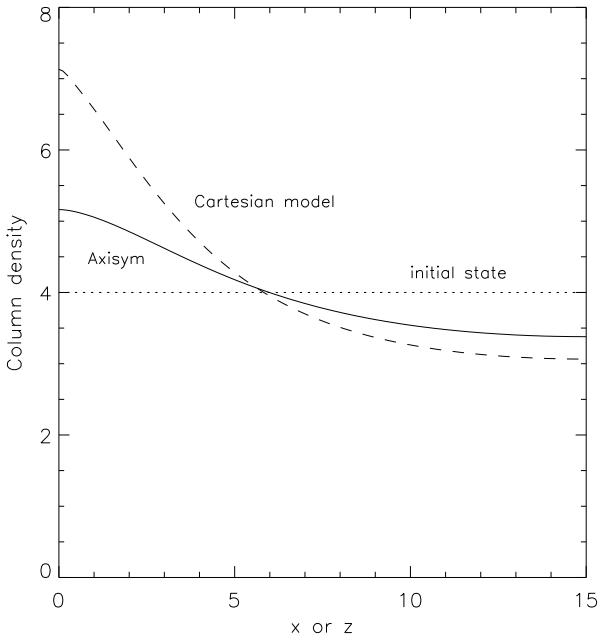

Fig. 8. Column density for the final states as a function of the horizontal coordinate for $\lambda/2 = 15$ and constant gravity. In the Cartesian model, the column density is $\Sigma(x) = \int_{-\infty}^{\infty} \rho(x, y)dy$. In the axisymmetric model, we plot the column density along a line of sight perpendicular to the axis of symmetry and with null impact parameter, i.e. $\Sigma(z) = \int_{-\infty}^{\infty} \rho(z, R)dR$. The column density at the initial states is drawn as a dotted line.

cal case, the variation of α in the final state relative to the initial values should be also smaller. In Figure 10, we plot the distribution of the magnetic-to-gas pressure ratio along $x = 0$ and $x = X_{\max}$ for the Cartesian model, and along the same cuts $z = 0$ and $z = Z_{\max}$ for the axisymmetric model in the final state, for a perturbation with half the wavelength of 15. Although the general shape of the curves is rather similar in both geometries, the α values in the range $10 < y < 22$ at $x = X_{\max}$ are almost a factor of 10 larger than in the axisymmetric model at the same region (i.e. $10 < R < 22$ and $z = Z_{\max}$).

4.2. *Non-uniform gravitational acceleration*

In an infinite self-gravitating isothermal filament, the gravitational acceleration g_R at large R decays as $1/R$, whereas it increases linearly at small R. Therefore, the assumption that g_R is constant is only valid at some intermediate range in R. To close our analysis, in this subsection we present the final equilibria for axially-symmetric configurations in a more realistic external gravitational field.

According to the results of Kim & Hong (1998) for a Cartesian model, the growth time is reduced by almost an order of magnitude and the length scale of the most unstable mode by factors of $4 - 8$ when a more realistic gravity model is used, because in such a realistic disk the magnetic field strength decreases rapidly with vertical distance which promotes the development of strong magnetic buoyancy. We expect, therefore, that the length and timescales of the Parker instability in a filament will be modified if a more realistic gravity model is adopted.

In order to have a model as self-consistent as possible, we will assume that the external gravitational potential is dominated by a collisionless (stellar or dark matter) component with isothermal one-dimensional velocity dispersion $\sigma_\star$. The gravitational potential created by an isothermal filament is given by

$$\psi(R) = 2\sigma_\star^2 \ln\left(1 + \frac{R^2}{8H_\star^2}\right), \qquad (31)$$

where $H_\star$ is a parameter that specifies the scale height of the collisionless component satisfying $4\pi G\rho_{0,\star}(0)H_\star^2 = \sigma_\star^2$, where $\rho_{0,\star}(0)$ is the density of collisionless matter at $R = 0$ (e.g., Ostriker 1964). As already anticipated, the gravitational acceleration $g_R(R)$ increases linearly with R at $R \ll \sqrt{8}H_\star$, reaches a maximum at $\sqrt{8}H_\star$ and decreases as $1/R$ at $R \gg \sqrt{8}H_\star$. Under this gravitational field, the density and magnetic field in the unperturbed state