TABLE 1

EMPIRICAL SERIES REPRODUCING THE HELIOCENTRIC DISTANCES OF THE PLANETS

MODIFIED TITIUS BODE LAW										
This Paper		This paper								
Body	TB	TBmod			a	T	a_p	T_p	Δa_n	ΔT_n
	N	X	X'	N'	AU	Year	AU	Year	%	%
Mercury	0	0	0	0	0.4	0.253	0.387	0.251	3.4	0.8
Venus	3	0	3	3	0.7	0.586	0.723	0.615	−3.2	−4.8
Earth	6	3	3	6	1.0	1.000	1.000	1.000	0.0	0.0
Mars	12	6	6	12	1.6	2.024	1.523	1.881	5.0	7.6
Ceres	24	12	12	24	2.8	4.685	2.765	4.600	1.2	1.9
Jupiter	48	24	24	48	5.2	11.858	5.202	11.862	−0.1	−0.0
Saturn	96	48	48	96	10.0	31.623	9.510	29.458	5.2	7.3
Uranus	192	96	96	192	19.6	86.773	19.25	84.013	1.8	3.3
Neptune		192	96	288	29.2	157.788	30.19	164.749	−3.3	−4.2
Pluto		288	96	384	38.8	241.684	39.50	247.492	−1.8	−2.3

TB is the original succession N defining the Titius-Bode law. TBmod is the modified TB law by means of the empirical succession X and X' and $N' = X + X'$. In Columns a and T we give the semi-major axes of orbits of the planets calculated by the TBmod model $(N' + 4)/10$ and the period computed by the way of the Third Law of Kepler, respectively. In Columns a_p and T_p we present the astronomical values of the observed orbital parameters. In the last two columns we give the determination of the error between the computed heliocentric distances and the astronomical ones Δa and ΔT, where the values tending to zero indicate a good degree of certainty.

planetary objects. Patton (1988) obtained the TB Law for Lagrange multipliers and applied it with acceptable results to eleven large asteroids. Through the approach of Monte Carlo, Lynch (2003) concluded that it is not possible to assure that the Titius-Bode Law explains the distribution of planets and satellites and Chang (2008) found that the distribution of the ratio of rotational periods in the 55 Cnc system is apparently inconsistent with that derived from the Titius-Bode Law. More recently Kotliarov (2009) has presented an interesting discussion of the Butusov model and proposed a generating function for the distribution of the planets.

2. MODIFIED TITIUS-BODE LAW

We propose a modification to the Titius-Bode Law (TBmod) by combining the sequences X (0, 0, 3, 6, 12, 24, 48, 96, 192, 288, 384, 576, 960, 1728) and X' (0, 3, 3, 6, 12, 24, 48, 96, 96, 96, 192, 384, 768, 1536), which were derived empirically by García Guerra (2005). The sum of the sequences $N' = X + X'$ reproduces one by one the terms of the succession of Titius-Bode (TB) from Mercury to Pluto, including the approach to the asteroid Ceres and other distant planetary objects. The semi-major axes of the planets are derived from the modified TB formulation $a = (N' + 4)/10$ which shows the curvilinear distribution of distances for each one of them (Figure 1). We presented the exponential function fit $r = e^{0.6122x-2.8176}$, $R^2 = 0.9990$ obtained from the interpolation ($r = Ae^{x/t}$) and we compared it with the semi-major axes of major planetary bodies and other small objects, among them Ceto, Eris, Sedna and the distant object SQ372.

In Table 1 we display the orbital parameters a and T calculated with the TBmod model, and the data for a_p and T_p (Astronomical Almanac 2010); in the columns $\Delta a = (a_n - a_p)/a_p$ (Kotliarov 2008) and $\Delta T = (T_n - T_p)/T_p$ we give the proportionality ratios between the calculated and observed values to measure the degree of certainty, which is considered acceptable if their values are close to zero. In our case these TBmod ratios have values of less than 5% and 8% respectively. One of our goals is to determine some kind of mathematical expression that would represent the general TBmod behavior, and to compare it with tendencies that show the different models deduced from the TB law summarized in Table 2.