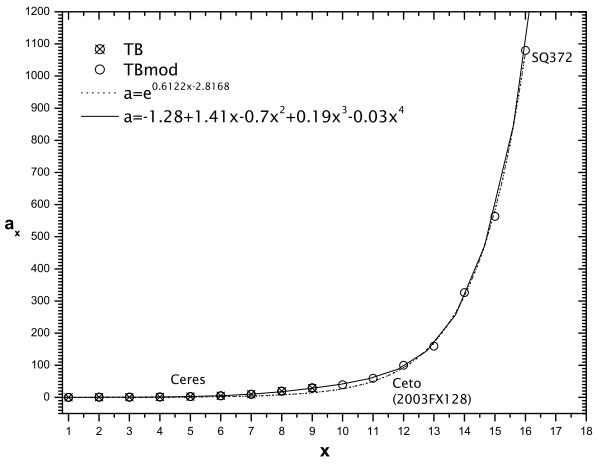

Fig. 1. Distribution of the semi-major axes of the planets and some small objects, given by the original and modified Titius-Bode Law, TB and TBmod respectively. We also present two interpolations that summarize the combination with an exponential function ($r = Ae^{x/t}$) and a seventh order polynomial (we only display the five first terms of the series), showing interpolated fits from Neptune to the interior region of the Kuiper Band.

Given the sequence of N' integer numbers; we carry out a setup of numerical operations in order to construct a function 2^n, which leads us to write this sequence like multiples of the common factor $3 \cdot 2^5$ whose first value $N' = 0$ for $n = 1$ (Mercury) we took into account in the discussion even though it does not exactly reproduce value the 0.387 of Mercury (Table 1). The resulting equations are:

$$r_n = 0.4 + 9.6 \cdot 2^{n-7}\,, \qquad n \leq 8 \qquad (1)$$

$$r_n = 29.2\,, \qquad n = 9 \qquad (2)$$

$$r_n = 29.2 + 9.6 \cdot \Sigma_{i=10}^{n}\ 2^{i-10}. \quad n \geq 10 \qquad (3)$$

If we arrange the elements $n = 9$ and $n \geq 9$ we realize that they constitute the total sum of the geometric progression given by $(2^{n-9} - 1)$, so that eventually we obtain the modified TBmod model, governed by two generic intervals of arithmetic and geometric progressions, respectively

$$r_n = \ 0.4 + 9.6 \cdot 2^{n-7}\,, \qquad n \leq 8 \qquad (4)$$

$$r_n = 19.6 + 9.6 \cdot 2^{n-9}\,, \qquad n \geq 9 \qquad (5)$$

Among the distributions of planetary bodies shown in Table 1 we observed that they present tendencies in the internal region of the Solar System including the TBmod distribution (Figure 2), although some of them diverge for the outer planets and even more so for distant objects. Note that all the models have difficulty in reproducing the parameters of Mercury, so that historically it has been necessary to postulate special conditions for this planet; we shall see how to avoid this problem.

TABLE 2

SOME MODELS THAT FIT THE TITIUS-BODE LAW

Titius-Bode Model	Reference
$r_n = 0.3 + 0.3 \cdot 2^n$	Wurm (1803)[a]
$n = -1, 0, 1, 2, ..., 6$	
$r_n = 0.2792 \cdot 1.53^n$	Armellini (1921)[a]
$n = 1, 2, 3, ..., 11$	
$r_n = 0.4 + 0.0075 \cdot 2^n$	Nicolini (1957)[a]
$n = -1, 0, 1, 2, ..., 6$	
$r_n = 0.2792 \cdot 1.52^n$	Munini & Armellini (1978)[a]
$n = 1, 2, 3, ..., 11$	
$r_n = 0.283 \cdot 1.52^n$	Badolati (1982)
$n = 1, 2, ...$	
$r_n = 0.4 + 0.3 \cdot 2^{(n-2)}$	Holton & Brush (2001)
$n = -\infty, 0, 1, 2, ..., 9$	
$r_n = e^{0.53074n - 1.51937}$	Ortiz et al. (2007)
$n = 0, 1, 2, 3, 4, ...$	
$r_n = e^{0.5894n - 1.65026}$	Poveda & Lara (2008)
$n = -\infty, 0, 1, 2, ..., 9$	
$r_n = e^{0.53707n - 1.661}$	Pankovic & Radakovic (2009)
$n = -1, 0, 1, 2, ..., 6$	

[a]As cited by Badolati 1982.

The actual distribution of the planets is contained in the TBmod curvilinear path, which coincides with the values of observed semi-major axes. We have chosen the logarithmic scale to better visualize the linear trace of the other six models. The Armellini and Bodolati model does not reproduce the distribution of planetary objects beyond Jupiter. The Nicolini and Holton & Brush models match the TBmod up to Uranus, as the Nicolini model is only for the planets. The Armellini and Bodolati models show values a_n that are very close. The case of the models of Wurm (not shown in Figure 2) falls short of all values Δa_n. In Ortiz et al. (2007), Poveda & Lara (2008), and Pankovic & Radakovic (2009) we observe that their traces exhibit a certain parallelism and proximity between the distribution of planetary objects. Finally, these cases including TBmod are those that best approximate the actual distribution of planetary objects. Of course the TBmod proposal shows the best approximation to the above mentioned distribution.